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Pricing uncertainty in stochastic multi-stage electricity markets

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Abstract— **This work proposes a pricing mechanism for multi-stage electricity markets that does not explicitly depend on the choice of dispatch procedure or optimization method. Our approach is applicable to a wide range of methodologies for the economic dispatch of power systems under uncertainty, including multi-interval dispatch, multi-settlement markets, scenario-based dispatch, and chance-constrained dispatch policies. We prove that our pricing scheme provides both *ex-ante* and *ex-post* dispatch-following incentives by simultaneously supporting per-stage and ex-post competitive equilibria. In numerical experiments on a ramp-constrained test system, we demonstrate the benefits of scheduling under uncertainty and show how our price decomposes into components corresponding to energy, intertemporal coupling, and uncertainty.**

I. Introduction

Rapid changes in the composition of the generation mix in power markets is creating several challenges for system operators (SOs). First, increasing renewable penetration from solar and wind is injecting variability and uncertainty into available power supply. Second, there is a lack of suitable market mechanisms tailored to the physical characteristics of DERs (such as energy storage) which are seeking to join markets in increasing numbers. Third, electrification of vehicle charging and thermal (heating/cooling) loads is impacting the shape and variability of the demand profile, leading to periods of high, sustained ramping.

These factors have a common theme of uncertainty, and SOs have been rapidly innovating on new market structures and dispatch procedures to handle it. These include multi-interval lookahead dispatch [1], ramping reserves [2], operating reserves [3], capacity markets [4], and multi-stage or intraday markets. Alongside, researchers have been investigating techniques from stochastic optimization to efficiently dispatch the market under uncertainty, including robust optimization [5], [6], chance-constrained optimization [7], scenario optimization [8], and distributionally robust optimization [9].

Uncertainty impacts the stability of pricing signals and can lead to market distortions such as out-of-merit dispatch, ramping shortages, and load shedding. Even with more advanced and accurate forecasts, SOs must still dispatch the system in a way that anticipates forecast uncertainty and the possibility of distribution shift over time. Pricing that incorporates characterizations of uncertainty is necessary to fairly and efficiently compensate different resources for their contributions to a reliable power supply.

The contribution in this paper is a pricing scheme for multi-stage markets that does not depend on the particular characterization of uncertainty or the method for optimizing over dispatch decisions that account for this uncertainty. Our approach is different from those in several recent works where the construction of the energy price intimately depends on the optimization paradigm (e.g., chance-constrained [9], robust [6], or rolling-window [10], [11]). We show that our proposed prices can be decomposed into components corresponding to the standard locational marginal price (LMP), intertemporal coupling, and uncertainty. Finally, we establish that this price clears the market under profit-maximizing assumptions on the participants and that it supports both *ex-ante* and *ex-post* dispatch-following incentives (see Section III-B for definitions).

A. Related Work

Our work draws on two main lines of inquiry into electricity market mechanism design. The first is dispatching and pricing multi-interval markets in the presence of intertemporal coupling constraints. The second is dispatching and pricing using techniques from robust and stochastic optimization.

a) Pricing multi-period electricity markets: In rolling-window real-time economic dispatch schemes, distribution shift in predicted net demand can lead to lost opportunity cost and distorted truthful bidding incentives for generators. Several pricing mechanisms building on standard uniform pricing schemes have been proposed in recent years to mitigate the lack of dispatch-following incentives [1], [12], [13]. A more recent line of work [10], [11], [14] has proposed a non-uniform pricing scheme, Temporal Locational Marginal Pricing (TLMP), and has established a dual definition of dispatch-following incentives. Simultaneously satisfying a "partial equilibrium" (i.e., *ex ante* dispatch-following incentive in every stage) and a general equilibrium (i.e., *ex post*) forms the notion of "strong equilibrium," used in this work.

Our pricing mechanism is distinguished from these works as they do not incorporate uncertainty directly in the lookahead dispatch algorithms, but rather design prices to mitigate incentive misalignment as a result of inaccurate predictions and distribution shift. However, these lookahead algorithms might be infeasible [15], [16], necessitating our development of more general pricing schemes that can incorporate such robust constraints.

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b) Pricing stochastic electricity markets: There has been much recent interest in designing electricity markets incorporating robust or stochastic constraints to ensure reliable operation in the face of uncertainty. For example, such dispatch schemes include economic dispatch with robust constraints [6], [5], chance constraints [17], [18], [19], distributionally robust chance constraints [9], [20], and conditional value at risk constraints [7]. However, in the subset of these works that explicitly address the problem of designing price mechanisms for the stochastic dispatch problem, inconsistent notions of *ex ante* dispatch-following incentives are considered which leaves open the need for out-of-market settlements to make up for lost opportunity cost.

This work improves practically upon existing methodologies by combining the temporally-coupled multi-interval dispatch used in practice with stochastic market-clearing mechanisms proposed in the research literature. Our approach can be applied to any formulation of stochastic or robust economic dispatch and ensures zero lost opportunity cost on the part of market participants by considering both *ex ante* and *ex post* dispatch-following incentives in the price specification.

II. Multi-stage dispatch under uncertainty

The day-ahead (DA) and real-time (RT) stages of electricity market clearing form a $T+1$ stage sequential optimization problem, with coupling between the stages and uncertainty from load and renewables realized between each of the T stages. The first stage is the single-shot, DA optimization problem which determines a unit commitment and associated dispatch for the upcoming 24-hour time horizon. This dispatch, although not physically realized, may be financially settled. Subsequently, in real time, a receding-horizon multi-interval optimization is performed. The first interval from each of these T subproblems is financially binding. Between each of the subproblems, the SO utilizes updated forecasts of uncertain demand and renewable generation to improve the efficiency of the dispatch.

The stages of the sequential problem are temporally coupled in the manner depicted in Figure 1. The first (DA) stage couples to all of the subsequent stages because it fixes the unit commitment – and therefore the upper/lower generation bounds, ramp limits, etc. – in the T subsequent (RT) stages. Within the RT market, stages are coupled consecutively due to the form of ramping constraints and the battery state-of-charge updates.

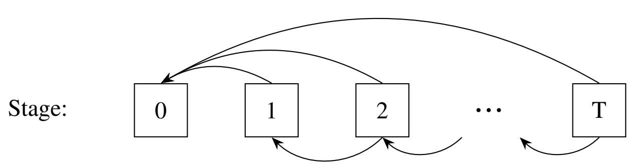

Fig. 1. Coupling between $T+1$ stages in DA + RT economic dispatch. A directed edge between two stages indicates that the later-stage decision depends explicitly on the decision committed to in the earlier stage.

Since the $T+1$ stages are solved and settled sequentially, we consider two groups of stages at a time: the period with no uncertainty, and the set of periods with remaining uncertainty. In the DA stage, the SO seeks to solve a stochastic optimization problem that fixes here-and-now decisions for the unit commitment while selecting policies for the wait-and-see decisions of RT stage 1. The purpose of the policies is to provide realization-dependent recourse in subsequent stages. However, in each of these stages, after uncertainty has been revealed, the multi-interval optimization is solved again for the next stage.

A. Notation

For each optimization interval indexed by $t \in \{0, \ldots, T\}$, each market participant $i \in \{1, \ldots, N\}$ has a dispatch vector $\mathbf{x}_{i,t} \in \mathbb{R}^{M_{i,t}}$ where $M_{i,t}$ is the dimension of the dispatch vector for i in stage t. The dispatch $\mathbf{x}_{i,t}$ includes all of the quantities associated with participant i in stage t. For conventional generators, this is just their power generation. For storage resources, it includes power generation and state-of-charge. We do not consider discrete variables, such as those needed for unit commitment, in our presentation here. They can be included without impacting our pricing or dispatch results, although the dispatch problem would need to be modified slightly as in [9], [21]. System states, such as nodal power injections, line flows, and voltage angles, can be written in terms of the individual dispatch variables $\mathbf{x}_{i,t}$ and are therefore not explicitly notated. For each t, we collect dispatch vectors into a single decision vector:

$$\mathbf{x}_t \coloneqq (\mathbf{x}_{1,t}, \ldots, \mathbf{x}_{N,t}) \in \mathbb{R}^{M_t},$$

where $M_t \coloneqq \sum_i M_{i,t}$. Associated with each dispatch vector is a market price $\boldsymbol{\pi}_t \in \mathbb{R}^{M_t}$. The revenue (or payment) each participant receives over the entire horizon is $\boldsymbol{\pi}_t^\top \mathbf{x}_t$.

For each t we associate a random vector of uncertainty $\boldsymbol{\xi}_t \in \mathbb{R}^{P_t}$. Realizations of $\boldsymbol{\xi}_t$, denoted $\hat{\boldsymbol{\xi}}_t$, are obtained sequentially after the dispatch $\hat{\mathbf{x}}_{t-1}$ has been committed but prior to computing $\mathbf{x}_t$. We also assume that the SO has access to a forecast $\boldsymbol{\theta}_t$ that represents their best knowledge at stage t about subsequent uncertainty $\boldsymbol{\xi}_{t+1}, \ldots, \boldsymbol{\xi}_T$. The composition of the forecast depends on what information is accessible. In the simplest case, $\boldsymbol{\theta}_t$ is just a point forecast of $\boldsymbol{\xi}_{t+1}, \ldots, \boldsymbol{\xi}_T$. When distributional information is available, $\boldsymbol{\theta}_t$ can be a set of parameters describing each forecast distribution and its support. Since stage 0 is the DA/UC stage of the market clearing, which happens when no uncertainty has been realized, $\hat{\boldsymbol{\xi}}_0$ is defined to be a set of forecasts over the subsequent T RT intervals.

In the rest of the paper, we denote by $\mathbf{a}_{\tau:t}$ the set of vectors $\{\mathbf{a}_j\}_{j=\tau}^t$. If $\tau > t$, we define this to be the empty set. For $\tau, t \in \mathbb{N}$ satisfying $\tau \leq t$, we define $[\tau, t] \coloneqq \{\tau, \tau+1, \ldots, t\}$.

B. Ex-post Dispatch Problem and Prices

If the SO had perfect forecasts of uncertainty, it could solve the following optimization problem (1) for all time intervals simultaneously. This is a useful solution because it benchmarks the efficiency of dispatch algorithms and quantifies the impact of uncertainty.

Problem 1. *Given an uncertainty realization $\hat{\boldsymbol{\xi}}$, the ex-post dispatch problem for all $T+1$ stages is:*

$$
\begin{aligned}
\min_{\mathbf{x}_0,\ldots,\mathbf{x}_T} \quad & \sum_{t=0}^{T}\sum_{i=1}^{N} c_{i,t}(\mathbf{x}_{i,0:t};\hat{\boldsymbol{\xi}}_{0:t}) && && \text{(1a)}\\
\text{s.t.} \quad & f_t(\mathbf{x}_t;\hat{\boldsymbol{\xi}}_{0:t}) \le \mathbf{0} && \forall t && \text{(1b)}\\
& g_{i,t}(\mathbf{x}_{i,t};\hat{\boldsymbol{\xi}}_{0:t}) \le \mathbf{0} && \forall i,\ \forall t && \text{(1c)}\\
& h_{i,t}(\mathbf{x}_{i,0:t};\hat{\boldsymbol{\xi}}_{0:t}) \le \mathbf{0} && \forall i,\ \forall t && \text{(1d)}
\end{aligned}
$$

Our formulation contains three types of constraints: (1b) convex system-wide constraints f_t that couple decisions across market participants but *within* each stage (e.g., power balance, line flow limits, zonal constraints, reserve requirements); (1c) private constraints $g_{i,t}$ for participant i and stage t (e.g., generation limits, state-of-charge (SOC) limits); and (1d) private constraints $h_{i,t}$ for participant i coupling their decisions in stage t to all previous dispatches (e.g., ramping, storage SOC updates, unit commitment-dependent generation limits).

This formulation of economic dispatch incorporates linear power flow equations, network constraints, zonal constraints, reserve constraints, private constraints, and intertemporal constraints for both conventional generators, flexible and inflexible loads, and storage.

Assumption 1. *We assume that for each i,t, functions $c_{i,t}$, f_t, $g_{i,t}$, and $h_{i,t}$ are convex w.r.t $\mathbf{x}_t$. We also assume that they are causal, in the sense that they possibly depend on any dispatches and uncertainty realized until time t. Finally, for non-triviality, we assume that Problem 1 has a feasible solution.*

If market dispatches $\mathbf{x}_0^*,\ldots,\mathbf{x}_T^*$ are generated by the optimal solution of Problem 1, then the market clearing price that supports a competitive equilibrium is just the dual multiplier associated with constraint (1b), cf. [13], [10].

C. Sequential Market Dispatch

In practice, solving Problem 1 is not a viable procedure for clearing the market due to the combination of uncertain inter-stage coupling constraints. Instead, SOs resort to solving a sequence of market-clearing optimization problems.[1] For each stage, updated forecasts of uncertainty are used as problem parameters, and advisory forward decisions are computed, but only the decision for the current stage is settled.

The market-clearing problem for stage t is presented in Problem 2, where the function $V_t : \mathbb{R}^{M_t} \to \mathbb{R}$ represents the forward cost of dispatch $\mathbf{x}_t$; we refer to this as the *forward value* or *cost-to-go* function. As with the functions in Problem 1, V_t may be parameterized by all uncertainty realized up to t, all previous dispatches, as well as forecasts of future uncertainty $\boldsymbol{\theta}_t$ that are available at time t:

$$V_t(\mathbf{x}_t;\hat{\mathbf{x}}_{i,0:t-1},\hat{\boldsymbol{\xi}}_{0:t},\boldsymbol{\theta}_t)$$

In service of simpler notation, we make this dependence on parameters implicit in the remainder of the manuscript and simply refer to $V_t(\mathbf{x}_t)$, except where an explicit reference to a particular parameter is necessary. In Section II-D, we remark on how V_t is already incorporated in market dispatch problems in practice as well as on the theoretical benefits of abstracting the forward cost of decisions in this way.

Problem 2. *Let $\hat{\mathbf{x}}_{0:t-1}$ be the sequence of dispatches committed prior to stage t and $\hat{\boldsymbol{\xi}}_{0:t}$ the uncertainty realized through stage t. The sequential dispatch problem for interval t is:*

$$
\begin{aligned}
\min_{\mathbf{x}_t} \quad & \sum_i c_{i,t}(\mathbf{x}_{i,t},\hat{\mathbf{x}}_{i,0:t-1};\hat{\boldsymbol{\xi}}_{0:t}) + V_t(\mathbf{x}_t) && && \text{(2a)}\\
\text{s.t.} \quad & \boldsymbol{\lambda}_t \perp\ f_t(\mathbf{x}_t;\hat{\boldsymbol{\xi}}_{0:t}) \le \mathbf{0} && && \text{(2b)}\\
& \boldsymbol{\mu}_{i,t} \perp\ g_{i,t}(\mathbf{x}_{i,t};\hat{\boldsymbol{\xi}}_{0:t}) \le \mathbf{0} && \forall i && \text{(2c)}\\
& \boldsymbol{\eta}_{i,t} \perp\ h_{i,t}(\mathbf{x}_{i,t},\hat{\mathbf{x}}_{i,0:t-1};\hat{\boldsymbol{\xi}}_{0:t}) \le \mathbf{0} && \forall i && \text{(2d)}
\end{aligned}
$$

The dual multipliers associated with each set of constraints are indicated to the left of each constraint (and followed by "$\perp$"). When $V_t(\mathbf{x}_t)$ is convex with respect to $\mathbf{x}_t$, and the convexity conditions from Assumption 1 hold, then (2) is a convex optimization problem.

The following algorithm specifies how the system operator clears and settles the market over the multi-stage scheduling horizon. Note that at each stage, the SO requires a scheme for deciding the prices $\boldsymbol{\pi}_t^*$ (see below).

Algorithm 1.

1) *The SO generates a DA uncertainty forecast $\hat{\boldsymbol{\xi}}_0$ and solves Problem 2 for $t=0$ to produce decisions $\mathbf{x}_0^*$ and prices $\boldsymbol{\pi}_0^*$.*
2) *For $t=1,\ldots,T$:*
   a) *Nature realizes uncertainty $\hat{\boldsymbol{\xi}}_t$;*
   b) *The SO solves Problem 2 to produce dispatches $\mathbf{x}_t^*$ and prices $\boldsymbol{\pi}_t^*$;*
   c) *Each participant realizes dispatch $\hat{\mathbf{x}}_t \coloneqq \mathbf{x}_t^*$ and settles with the SO i at ${\boldsymbol{\pi}_{i,t}^*}^\top \hat{\mathbf{x}}_{i,t}$.*

Assumption 2. *Solving Problem 2 iteratively for $t=0,\ldots,T$ produces a feasible sequence of dispatches. Note that such recursive feasibility is in general not guaranteed and may depend on the choice of V_t and $\boldsymbol{\theta}$; see [15], [16] for further consideration of these details.*

D. Specifying the cost-to-go function V_t

Depending on the parameterization of the uncertainty forecast $\boldsymbol{\theta}_t$ and the choice of the stochastic optimization model, the function V_t adopts different forms. We show below how several common stochastic paradigms fit into this framework. These encompass the multi-settlement and rolling-window

[1] For example, this sequence could be the combination of a day-ahead forward market followed by real-time adjustment market clearings every 15 minutes.

optimization procedures (with and without lookahead) used by SOs in practice as well as stochastic optimization formulations increasingly studied in the research literature.

1) Rolling dispatch without lookahead: This procedure is the traditional approach to dispatching the DA and RT markets, where each stage (or interval) is optimized without considering the forward consequences of the current dispatch. Thus, $\mathbf{x}_t$ is only coupled intertemporally to $\hat{\mathbf{x}}_{t-1}$ through the constraints (2e). In this case, $V_t \coloneqq 0$ for all $t = 0, \ldots, T$. As this is convex, Problem 2 is, therefore, convex and tractable.

2) Rolling dispatch with lookahead: To better handle uncertainty, SOs make use of forecasts and advisory decisions over a lookahead horizon of length $h > 1$. Exploiting lookahead predictions can increase the feasibility and ex-post optimality of the overall dispatch sequence since it allows for anticipating future ramp, unit commitment, and storage charge/discharge needs [22]. The forecast is a point forecast $\boldsymbol{\theta}_t = (\tilde{\boldsymbol{\xi}}_{t+1}, \ldots, \tilde{\boldsymbol{\xi}}_{t+h})$, available at time t, of the true uncertainties $\hat{\boldsymbol{\xi}}_{t+1}, \ldots, \hat{\boldsymbol{\xi}}_{t+h}$ to be realized.

$$
\begin{aligned}
&V_t(\mathbf{x}_t; \boldsymbol{\theta}_t) \coloneqq \\
&\min_{\mathbf{x}_{t+1}, \ldots, \mathbf{x}_{t+h}} \sum_{\tau=t+1}^{t+h} \sum_{i=1}^{N} c_{i,\tau}(\mathbf{x}_{i,t+1:\tau}, \hat{\mathbf{x}}_{i,0:t}; \hat{\boldsymbol{\xi}}_{0:t}, \tilde{\boldsymbol{\xi}}_{t+1:\tau}) && \text{(3a)} \\
&\text{s.t.} \quad f_\tau(\mathbf{x}_\tau; \hat{\boldsymbol{\xi}}_{0:t}, \tilde{\boldsymbol{\xi}}_{t+1:\tau}) \le \mathbf{0} \quad \forall \tau && \text{(3b)} \\
&\qquad g_{i,\tau}(\mathbf{x}_{i,\tau}; \hat{\boldsymbol{\xi}}_{0:t}, \tilde{\boldsymbol{\xi}}_{t+1:\tau}) \le \mathbf{0} \quad \forall i, \forall \tau && \text{(3c)} \\
&\qquad h_{i,\tau}(\mathbf{x}_{i,t+1:\tau}, \hat{\mathbf{x}}_{i,0:t}; \hat{\boldsymbol{\xi}}_{0:t}, \tilde{\boldsymbol{\xi}}_{t+1:\tau}) \le \mathbf{0} \quad \forall i, \forall \tau && \text{(3d)}
\end{aligned}
$$

In the above, $\forall \tau$ means $\tau \in [t+1, t+h]$. By convention, if (3) is infeasible, $V_t = +\infty$.

Proposition 1. *$V_t(\mathbf{x}_t; \boldsymbol{\theta}_t)$ in (3) is convex in $\mathbf{x}_t$.*

Although V_t in (3) is convex, it is not possible to write down a closed-form solution in general. However, (3) can be incorporated into the formulation of the problem (2), recovering the standard lookahead economic dispatch problem studied in [10], [13], which is a tractable convex optimization problem. Note that in a solution $\mathbf{x}_t, \mathbf{x}_{t+1}, \ldots, \mathbf{x}_{t+h}$ to (2) with V_t defined as (3), only the first dispatch $\mathbf{x}_t$ is binding for the purposes of Algorithm 1. The remaining dispatches $\mathbf{x}_{t+1}, \ldots, \mathbf{x}_{t+h}$ are advisory and are re-computed for each successive interval.

3) Chance-constrained optimization: Chance-constrained optimization has received increasing interest for its ability to optimize over decisions with constraints involving stochastic uncertainty [17], [9], [23]. The form of V_t presented next enables probabilistic guarantees on the feasibility of the advisory dispatch under a distributional assumption on uncertainty. At time t, we define p_t to be the distribution of future uncertainty $\boldsymbol{\xi}_{t+1:t+h}$ conditioned on all uncertainty realizations through time t. The forecast $\boldsymbol{\theta}_t$ collects parameters of this distribution or of the SO's best guess of this distribution. In this case, the risk-neutral chance-constrained lookahead value function is defined as follows:

$$
\begin{aligned}
&V(\mathbf{x}_t; \boldsymbol{\theta}_t) \coloneqq \\
&\min_{\mathbf{x}_{t+1}, \ldots, \mathbf{x}_{t+h}} \mathop{\mathbb{E}}_{\boldsymbol{\xi}_{t+1:t+h} \sim p_t} \left[\sum_{\tau=t+1}^{t+h} \sum_{i=1}^{N} c_{i,\tau}(\mathbf{x}_{i,\tau}; \boldsymbol{\xi}_\tau) \right] && \text{(4a)} \\
&\text{s.t.} \quad \mathbb{P}_{\boldsymbol{\xi}_{t+1:\tau}}[f_\tau(\mathbf{x}_\tau; \hat{\boldsymbol{\xi}}_{0:t}, \boldsymbol{\xi}_{t+1:\tau}) \le \mathbf{0}] \ge 1 - \varepsilon_\tau^f \quad \forall \tau && \text{(4b)} \\
&\qquad \mathbb{P}_{\boldsymbol{\xi}_{t+1:\tau}}[g_{i,\tau}(\mathbf{x}_{i,\tau}; \hat{\boldsymbol{\xi}}_{0:t}, \boldsymbol{\xi}_{t+1:\tau}) \le \mathbf{0}] \ge 1 - \varepsilon_{i,\tau}^g \quad \forall i, \forall \tau && \text{(4c)} \\
&\qquad \mathbb{P}_{\boldsymbol{\xi}_{t+1:\tau}}[h_{i,\tau}(\mathbf{x}_{i,t+1:\tau}, \hat{\mathbf{x}}_{i,0:t}; \hat{\boldsymbol{\xi}}_{0:t}, \boldsymbol{\xi}_{t+1:\tau}) \le \mathbf{0}] \\
&\qquad\qquad \ge 1 - \varepsilon_{i,\tau}^h \quad \forall i, \forall \tau && \text{(4d)}
\end{aligned}
$$

In the above, $\forall \tau$ means $\tau \in [t+1, t+h]$. By convention, if (3) is infeasible, $V_t = +\infty$. The hyperparameter ε's can be tuned by the SO to adjust the permissible probability of a constraint violation.

In general, (4) is intractable due to the difficulty in computing probabilities and expectations over arbitrary distributions p_t. In particular, the feasible set defined by the constraints may be nonconvex even if the constraint functions $f_\tau, g_{i,\tau}, h_{i,\tau}$ are convex. The structure of the constraints may also make the problem infeasible, e.g., a fixed advisory decision will generally be insufficient to guarantee feasibility under any demand realization, and uncertainty-dependent recourse will be necessary. However, by introducing suitable assumptions on the structure of the problem such as linearity of $c_{i,\tau}, f_\tau, g_{i,\tau}, h_{i,\tau}$, Gaussianity of p_t, separating joint chance constraints into individual chance constraints, and replacing advisory decisions with advisory uncertainty-dependent affine policies, a tractable, convex counterpart to (4) can be formed. For details on such a transformation, we refer the reader to recent literature on chance-constrained optimization and economic dispatch [24], [17].

4) Other stochastic formulations: The procedure we have been following in Subsections II-D.1 – II-D.3 to formulate the sequential dispatch problem in the form (2) be applied to other stochastic optimization settings, including scenario-based optimization, robust optimization, and distributionally robust optimization, where there is an extensive literature on convex, tractable reformulations [20], [9], [8], [5], [16].

In fact, although all of these approaches to defining V_t rely on constructing a tractable optimization problem, this is not necessary for Problem 2. As long as V_t is convex and it is possible to obtain gradients of V_t for any input $\mathbf{x}_t$, then optimization (2) can be solved using gradient-based methods. And, as we will show in the next section, the price formation also depends only on being able to compute gradients of V_t for the market dispatch.

III. Pricing multi-stage uncertainty

In this section, we define the market clearing price and prove that it supports a competitive market clearing solution under *ex-ante* and *ex-post* definitions of dispatch-following incentives.

A. Model of market participation

In order to establish the properties of a competitive equilibrium, we first present the participant's model of market behavior. We assume that the agents are price-takers, in that they do not bid strategically to impact the price. Further, we assume that they optimize for the current stage of the optimization problem and do not price future decisions into the bid for the current interval. We express the agent's profit-maximizing behavior precisely through the following problem.

Problem 3. *Under a given price $\boldsymbol{\pi}_{i,t}$, agent i's profit-maximizing schedule in interval t is:*

$$\arg\max_{\overline{\mathbf{x}}_{i,t}} \quad \boldsymbol{\pi}_{i,t}^{\top}\overline{\mathbf{x}}_{i,t} - c_{i,t}(\overline{\mathbf{x}}_{i,t}, \hat{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_{0:t}) \tag{5a}$$

$$\text{s.t.} \quad \overline{\boldsymbol{\mu}}_{i,t} \perp g_{i,t}(\overline{\mathbf{x}}_{i,t}; \hat{\boldsymbol{\xi}}_{0:t}) \leq \mathbf{0} \tag{5b}$$

$$\overline{\boldsymbol{\eta}}_{i,t} \perp h_{i,t}(\overline{\mathbf{x}}_{i,t}; \hat{\mathbf{x}}_{i,0:t-1}, \hat{\boldsymbol{\xi}}_{0:t}) \leq \mathbf{0} \tag{5c}$$

B. Equilibrium Concepts

We are interested in pricing mechanisms that the SO can implement to promote *dispatch-following incentives*. These incentives come in two varieties: *ex-ante*, which apply before uncertainty realization and dispatch, and *ex-post*, which apply after uncertainty has been realized and dispatches have been committed. Adopting terminology from [10], [11], we now present equilibrium notions that will encourage both ex-ante and ex-post dispatch following incentives.

Definition 1. *Let $\mathbf{x}_0, \dots, \mathbf{x}_T$ be a dispatch sequence and $\boldsymbol{\pi}_0, \dots, \boldsymbol{\pi}_T$ be a price sequence, and let $\hat{\boldsymbol{\xi}}$ be a realization of uncertainty. This pair of sequences supports a* ***general equilibrium*** *over the entire scheduling horizon $t = 0, \dots, T$ if and only if the following conditions hold:*

1) *Market Clearing Condition. The dispatch sequence satisfies the system-wide constraints at all times:*

$$f_t(\mathbf{x}_t, \hat{\boldsymbol{\xi}}_{0:t}) \leq \mathbf{0} \qquad \forall t \in [0, T]$$

2) *Incentive Compatibility. For each participant i, $\mathbf{x}_{i,0}, \dots, \mathbf{x}_{i,T}$ is an optimal solution of the participant's* ex post *problem:*

$$\arg\max_{\overline{\mathbf{x}}_{i,0},\dots,\overline{\mathbf{x}}_{i,T}} \quad \sum_{t=0}^{T} \boldsymbol{\pi}_{i,t}{}^{\top}\overline{\mathbf{x}}_{i,t} - c_{i,t}(\overline{\mathbf{x}}_{i,t}, \overline{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_{0:t}) \tag{6a}$$

$$\text{s.t.} \quad \overline{\boldsymbol{\mu}}_{i,t} \perp g_{i,t}(\overline{\mathbf{x}}_{i,t}; \hat{\boldsymbol{\xi}}_{0:t}) \leq \mathbf{0} \qquad \forall t \in [0,T] \tag{6b}$$

$$\overline{\boldsymbol{\eta}}_{i,t} \perp h_{i,t}(\overline{\mathbf{x}}_{i,t}, \overline{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_{0:t}) \leq \mathbf{0} \qquad \forall t \in [0,T] \tag{6c}$$

A dispatch and price sequence that supports a general equilibrium supports ex-post dispatch-following incentives. However, when the SO schedules in the presence of uncertainty, e.g. in the case of multi-interval lookahead or stochastic dispatch, a missing payments problem can arise due to distribution shift. The works [10], [11] discuss this issue extensively in the lookahead setting and further show how this missing payment problem arises even when there are perfect forecasts (but a truncated lookahead horizon). To address this, they introduce an additional notion of *partial equilibrium* at each dispatch stage which may be viewed as a condition on *ex-ante* dispatch-following incentives.

Definition 2. *Let $\mathbf{x}_t$ be the dispatch and $\boldsymbol{\pi}_t$ be the price from interval t, and let $\hat{\boldsymbol{\xi}}_{0:t}$ be a realization of uncertainty up through t. This pair supports a* ***partial equilibrium*** *for stage t if and only if the following conditions hold:*

1) *Market Clearing Condition:*

$$f_t(\mathbf{x}_t, \hat{\boldsymbol{\xi}}_{0:t}) \leq \mathbf{0}$$

2) *Incentive Compatibility: For each i, the subvector $\mathbf{x}_{i,t}$ of $\mathbf{x}_t$ is the optimal solution of (5) under price $\boldsymbol{\pi}_{i,t}$.*

The work in [10], [11] also adopts a dual notion of equilibrium that combines partial and general equilibrium.

Definition 3. *A dispatch sequence $\mathbf{x}_0, \dots, \mathbf{x}_T$ and price sequence $\boldsymbol{\pi}_0, \dots, \boldsymbol{\pi}_T$ support a* ***strong equilibrium*** *under sequentially realized uncertainty $\hat{\boldsymbol{\xi}}_1, \dots, \hat{\boldsymbol{\xi}}_T$ if and only if they support both a general equilibrium and a partial equilibrium for each t.*

By employing this stronger notion of equilibrium, both ex-ante and ex-post incentive alignment can be guaranteed in the lookahead dispatch setting. We adopt this notion of strong equilibrium in our work to enable pricing that guarantees dispatch-following incentives in the case of general lookahead value function V_t, such as those in the case of stochastic optimization formulations of the market dispatch problem.

C. Pricing a strong equilibrium

In each interval, the market operator solves (2) to generate a dispatch for that interval for each participant $\mathbf{x}_{i,t}^*$ along with a price vector $\boldsymbol{\pi}_{i,t}^*$ defined as

$$\begin{aligned}\boldsymbol{\pi}_{i,t}^* := & \underbrace{-\mathsf{D}_{\mathbf{x}_{i,t}} f_t(\mathbf{x}_t^*; \hat{\boldsymbol{\xi}}_{0:t})^{\top} \boldsymbol{\lambda}_t^*}_{\text{Locational marginal price}} \underbrace{- \nabla_{\mathbf{x}_{i,t}} V_t(\mathbf{x}_{i,t}^*; \boldsymbol{\theta}_t)}_{\text{Price of uncertainty}} \\ & \underbrace{-\mathsf{D}_{\mathbf{x}_{i,t}} h_{i,t}(\mathbf{x}_{i,t}^*, \hat{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_{0:t})^{\top} \boldsymbol{\eta}_{i,t}^*}_{\text{Price of intertemporal coupling}}\end{aligned} \tag{7}$$

This price is defined in terms of optimal dual variables and derivatives of objective/constraint functions at the optimal point. The notation $\mathsf{D}_{\mathbf{x}_{i,t}} f_t(\mathbf{x}_t^*; \hat{\boldsymbol{\xi}}_t)$ represents the Jacobian of the function f_t with respect to variable $\mathbf{x}_{i,t}$ evaluated at $\mathbf{x}_{i,t} = \mathbf{x}_{i,t}^*$.

Our price admits a straightforward decomposition into several functional parts. The first component of the price is the standard locational marginal price (LMP). The second term prices the cost of scheduling under uncertainty. The magnitude of this term is determined both by the particular choice of V_t as well as the quality of the uncertainty parameterization in $\boldsymbol{\theta}_t$. The last component is a price on the intertemporal coupling between decisions. The price of ramping presented in [10] is a special case of this term; our formulation admits other intertemporal couplings, such as from storage state-of-charge [14]. This price is discriminatory, in that each participant may see a different price. The necessity of such price discrimination when there are intertemporal coupling constraints on generators is proven in [10].

We now establish the equilibrium properties of this price. Given the prior convexity assumptions on $c_{i,t}$, f_t, $g_{i,t}$, and $h_{i,t}$, problems (5) and (6) are convex.

Theorem 1. *Fix a $t \in [0,T]$ and let $\mathbf{x}_t^*$ be the dispatch produced by the optimal solution of (2) and let $\boldsymbol{\pi}_t^*$ be the price as defined in (7) using optimal primal/dual variables from (2). This dispatch-price pair forms a partial equilibrium for interval t.*

Proof. For an interval t, we have realized uncertainty $\hat{\boldsymbol{\xi}}_t$ and a previous dispatch sequence $\hat{\mathbf{x}}_{0:t-1}$. Assume that problem (2) has been solved to optimality yielding optimal primal/dual solutions (not necessarily unique) $\mathbf{x}_t^*, \boldsymbol{\lambda}_t^*, \boldsymbol{\mu}_{i,t}^*, \boldsymbol{\eta}_{i,t}^* \ \forall i$.

The market clearing condition in Definition 2 is satisfied by primal feasibility of the optimal solution $\mathbf{x}_t^*$. Without loss of generality, the rest of the proof will be shown for a particular i. To show incentive compatibility, we write down the Lagrangian of (2) for a given t:

$$\begin{aligned}\mathcal{L}_t = &\sum_{i=1}^{N} c_{i,t}(\mathbf{x}_{i,t}, \hat{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_{0:t}) + V_t(\mathbf{x}_t; \boldsymbol{\theta}_t) + \boldsymbol{\lambda}_t^\top f_t(\mathbf{x}_t; \hat{\boldsymbol{\xi}}_{0:t}) \\ &+ \sum_{i=1}^{N} \boldsymbol{\mu}_{i,t}^\top g_{i,t}(\mathbf{x}_{i,t}; \hat{\boldsymbol{\xi}}_{0:t}) + \sum_{i=1}^{N} \boldsymbol{\eta}_{i,t}^\top h_{i,t}(\mathbf{x}_{i,t}, \hat{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_{0:t})\end{aligned}$$

The stationarity conditions hold at optimality:

$$\begin{aligned}\mathbf{0} = &\nabla_{\mathbf{x}_{i,t}} c_{i,t}(\mathbf{x}_{i,t}^*, \hat{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_{0:t}) + \nabla_{\mathbf{x}_{i,t}} V(\mathbf{x}_t^*; \theta_t) \\ &+ \mathsf{D}_{\mathbf{x}_{i,t}} f_t(\mathbf{x}_t^*; \hat{\boldsymbol{\xi}}_{0:t})^\top \boldsymbol{\lambda}_t^* + \mathsf{D}_{\mathbf{x}_{i,t}} g_{i,t}(\mathbf{x}_{i,t}^*; \hat{\boldsymbol{\xi}}_{0:t})^\top \boldsymbol{\mu}_{i,t}^* \\ &+ \mathsf{D}_{\mathbf{x}_{i,t}} h_{i,t}(\mathbf{x}_{i,t}^*, \hat{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_{0:t})^\top \boldsymbol{\eta}_{i,t}^*\end{aligned} \tag{8}$$

The argument uses the convex KKT theorem. We construct primal-dual solutions that satisfy the KKT optimality conditions (primal/dual feasibility, complementary slackness, and stationarity) of problem (5). Because (5) is convex, the constructed primal-dual solution is also optimal. Define

$$\overline{\mathbf{x}}_{i,t} := \mathbf{x}_{i,t}^* \tag{9a}$$
$$\overline{\boldsymbol{\mu}}_{i,t} := \boldsymbol{\mu}_{i,t}^* \tag{9b}$$
$$\overline{\boldsymbol{\eta}}_{i,t} := \mathbf{0} \tag{9c}$$

$\overline{\mathbf{x}}_{i,t}$ satisfies primal feasibility of (5) because $\mathbf{x}_{i,t}^*$ is primal feasible for (2). $\overline{\boldsymbol{\mu}}_{i,t}$ and $\overline{\boldsymbol{\eta}}_{i,t}$ are dual feasible because both are non-negative by construction. Complementary slackness holds for $\overline{\boldsymbol{\mu}}_{i,t}$ because $\boldsymbol{\mu}_{i,t}^*$ is optimal for (2), and holds for $\overline{\boldsymbol{\eta}}_{i,t}$ trivially.

The Lagrangian of (5) is

$$\begin{aligned}\mathcal{L}_{i,t} = &-\boldsymbol{\pi}_{i,t}^{*\top} \overline{\mathbf{x}}_{i,t} + c_{i,t}(\overline{\mathbf{x}}_{i,t}, \hat{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_{0:t}) \\ &+ \overline{\boldsymbol{\mu}}_{i,t}^\top g_{i,t}(\overline{\mathbf{x}}_{i,t}; \hat{\boldsymbol{\xi}}_{0:t}) + \overline{\boldsymbol{\eta}}_{i,t}^\top h_{i,t}(\overline{\mathbf{x}}_{i,t}, \hat{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_{0:t})\end{aligned} \tag{10}$$

Now to check the stationarity condition,

$$\begin{aligned}\nabla_{\overline{\mathbf{x}}_{i,t}} \mathcal{L}_{i,t} &= -\boldsymbol{\pi}_{i,t}^* + \nabla_{\overline{\mathbf{x}}_{i,t}} c_{i,t}(\mathbf{x}_{i,t}^*, \hat{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_t) \\ &\quad + \mathsf{D}_{\overline{\mathbf{x}}_{i,t}} g_{i,t}(\mathbf{x}_{i,t}^*; \hat{\boldsymbol{\xi}}_{0:t})^\top \boldsymbol{\mu}_{i,t}^* + \mathbf{0} \\ &= \mathsf{D}_{\mathbf{x}_{i,t}} f_t(\mathbf{x}_t^*; \hat{\boldsymbol{\xi}}_{0:t})^\top \boldsymbol{\lambda}_t^* + \nabla_{\mathbf{x}_{i,t}} V(\mathbf{x}_t^*; \theta_t) \\ &\quad + \mathsf{D}_{\mathbf{x}_{i,t}} h_{i,t}(\mathbf{x}_{i,t}^*; \hat{\mathbf{x}}_{i,t-1}, \hat{\boldsymbol{\xi}}_t)^\top \boldsymbol{\eta}_{i,t}^* \\ &\quad + \nabla_{\overline{\mathbf{x}}_{i,t}} c_{i,t}(\mathbf{x}_{i,t}^*, \hat{\mathbf{x}}_{i,0:t-1}; \hat{\boldsymbol{\xi}}_t) \\ &\quad + \mathsf{D}_{\overline{\mathbf{x}}_{i,t}} g_{i,t}(\mathbf{x}_{i,t}^*; \hat{\boldsymbol{\xi}}_{0:t})^\top \boldsymbol{\mu}_{i,t}^* \\ &= \mathbf{0}\end{aligned}$$

where the first equality comes by from plugging (9) into (10) and the second equality comes from plugging in the price defined in (7). The third equality holds because the expression is identical to (8). ☐

Theorem 2. *The sequences of dispatches $\mathbf{x}_0^*, \ldots, \mathbf{x}_T^*$ and prices $\boldsymbol{\pi}_0^*, \ldots, \boldsymbol{\pi}_T^*$ produced by Algorithm 1 over the entire scheduling horizon form a general equilibrium.*

We omit the proof due to space constraints. The approach is the same as for Theorem 1, where we construct a primal-dual solution for (6) and show that the dispatches generated by Algorithm 1 and prices guarantee its optimality. The intertemporal coupling and uncertainty terms allow the Lagrangian of (6) to decouple across intervals and thus the optimality conditions of (2) apply simultaneously.

The result in Theorem 2 shows that price (7) guarantees that each participant has zero lost opportunity cost at the end of the scheduling horizon. The intertemporal coupling term compensates participants for any lost opportunity cost due to binding intertemporal constraints (e.g., ramping) whereas the uncertainty term compensates participants for any lost opportunity cost due to the system operator's uncertainty-aware scheduling procedure.

The following corollary holds from Theorems 1 and 2:

Corollary 1. *The sequences of dispatches $\mathbf{x}_0^*, \ldots, \mathbf{x}_T^*$ and prices $\boldsymbol{\pi}_0^*, \ldots, \boldsymbol{\pi}_T^*$ produced by Algorithm 1 over the entire scheduling horizon support a strong equilibrium.*

A strong equilibrium is a desirable property of a market-clearing price because it provides dispatch-following incentives during each stage of scheduling horizon while also correcting the missing payment problem that arises ex-post.

IV. Experiments

We explore how uncertainty affects dispatch efficiency and pricing under our mechanism through a test case similar to that presented in [11]. We consider a power system with a gas combined-cycle (C.C.) plant, a gas peaker plant, solar, wind, and load in a single bus network. The gas plants are ramp constrained whereas the renewables are not. Cost functions are linear and are parameterized by their marginal cost. All parameters for the generators are given in Table I.

TABLE I

GENERATOR PARAMETERS FOR THE TEST CASE

Generator	Pmin (MW)	Pmax (MW)	Ramp Rate (% Pmax/hour)	Cost ($/MWh)
Gas C.C.	350	550	25%	50
Gas Peaker	100	120	200%	70
Solar	0	250	NA	0
Wind	0	350	NA	0

We obtain 24-hr load and renewable generation profiles from CAISO from Sep. 9, 2021 [25]. These include both forecast day-ahead trajectories and the actual, realized real-time trajectories, all of which were normalized to 1000MW peak demand. Sample realizations of the true trajectories are simulated by adding correlated zero-mean Gaussian noise to the actual values.

Algorithm 1 was implemented to clear the market in a rolling fashion. The dispatch horizon for a single run of the market is 24 hours, consisting of 289 individual stages: one DA dispatch and RT dispatches every 5 minutes. The first stage ($t = 0$) is the DA unit commitment problem. The unit commitment problem makes use of a 24-hour ahead hourly DA forecast in the CAISO data.[2] The subsequent RT stages take the unit commitment as fixed.

We implement the three mechanisms from Section II-D for dispatching in RT. First is myopic scheduling, where only the current interval's cost and constraints are optimized but generator ramping constraints bind the current decision to the realized dispatch from the previous interval. This is a deterministic problem, as demand and renewable generation are assumed to be known, and does not account for the cost of future decisions in the scheduling horizon. Second is multi-interval lookahead scheduling with a 3-hour lookahead horizon.A lookahead forecast is computed by taking the mean of a small subset of random trajectories. Third is a multi-interval chance-constrained lookahead problem, where the constraints for the advisory periods hold probabilistically and the objective function is the expected cost for the advisory periods. The distribution parameters of the trajectory forecasts are obtained from the set of randomly sampled trajectories.

Figure 2 shows the dispatch trajectories for each of the generators in the system under optimal *ex-post* scheduling. Note that due to its high cost relative to the other generators, the gas peaker is only active during the peak demand hours when the ramp needs of the system exceed available capacity. Lookahead dispatch with point forecasts results dispatching the peaker less often for binding ramping constraints but more during other intervals due to the cost of uncertain dispatch. Chance-constrained lookahead dispatch is able to avoid most of the binding ramping constraints at the expense of more

[2] In North American ISOs, there is often a financial settlement in the DA market. Although our formulations accommodate a financially settled DA market, we do not empirically analyze the DA market settlement in this work, as intertemporal coupling and uncertainty do not arise in the formation of the DA prices.

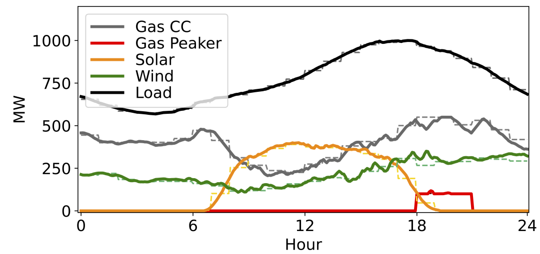

Fig. 2. DA (dashed line) and optimal RT (solid line) dispatch trajectories for generators and load over a 24 hour scheduling horizon.

precautionary dispatches due to uncertainty.

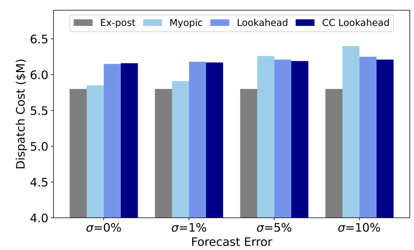

Fig. 3. Total dispatch cost of the different pricing schemes under increasing forecast error. Forecast error is defined as the mean absolute percentage deviation from the true trajectory realization.

Figure 3 presents the benefits of scheduling with lookahead and stochastic forward cost policies. When the forecast error is zero, accounting for forward cost results in more costly dispatch than myopic scheduling. This is due to the inherent conservatism and robustness that these policies provide. However, as uncertainty increases, myopic scheduling becomes more costly than uncertainty-aware scheduling due to load shedding actions and sub-optimal dispatch of higher cost generators.

Finally, we show how our proposed market clearing price (7) decomposes into its constituent components in Figure 4. The largest component of the price is the uniform shadow price of the power balance constraint. However, for the ramp-constrained gas generator, there are additional terms that compensate for the opportunity cost of the system operator's imperfect scheduling under uncertainty.

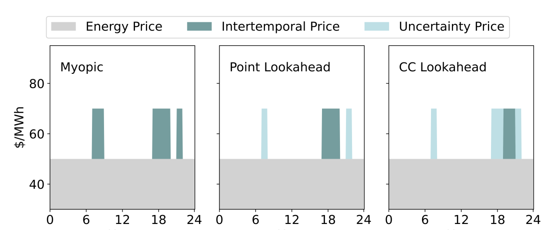

Fig. 4. Price trajectory $\pi^*_{i,t}$ for the gas combined-cycle generator under $\sigma = 10\%$ forecast uncertainty for different real-time forecast methodologies.

The additional complexity of computing price (7) is negligible compared to the standard LMP and T-LMP in [10], as it is also defined in terms of optimal dual variables and cost function gradients. The complexity of the dispatch problem depends on the choice of procedure (e.g., change-constrained, robust, scenario).

V. Conclusion

In this paper, we have presented a unified mechanism for pricing uncertainty in a multi-stage dispatch setting, incorporating both standard deterministic lookahead dispatch and stochastic market clearing approaches (e.g., chance-constrained, robust) within the same pricing framework. We prove that our price provides dispatch following incentives as well as zero lost opportunity costs for generators. A detailed empirical comparison with other pricing methodologies, such as the standard LMP and the R-TLMP [10], is reserved for future work. Ongoing research analyzes the system operator's merchandizing surplus and compares multi-settlement and single-settlement pricing methodologies.

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