

# On the Interactions Between Multiple Overlapping WLANs Using Channel Bonding

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**Abstract**—Next-generation wireless local area networks (WLANs) will support the use of wider channels, which is known as channel bonding, to achieve higher throughput. However, because both the channel center frequency and the channel width are autonomously selected by each WLAN, the use of wider channels may also increase the competition with other WLANs operating in the same area for the available channel resources. In this paper, we analyze the interactions between a group of neighboring WLANs that use channel bonding and evaluate the impact of those interactions on the achievable throughput. A continuous-time Markov network model that is able to capture the coupled dynamics of a group of overlapping WLANs is introduced and validated. The results show that the use of channel bonding can provide significant performance gains, even in scenarios with a high density of WLANs, although it may also cause unfair situations in which some WLANs receive most of the transmission opportunities while others starve.

**Index Terms**—Carrier sense multiple access with collision avoidance (CSMA/CA), channel allocation, channel bonding, dense networks, IEEE 802.11ac, IEEE 802.11ax, wireless local area networks (WLANs).

## I. INTRODUCTION

THE number of multimedia devices, including smartphones, laptops, and high-definition audio/video players, that access the Internet through deployed wireless local area network (WLAN) access points is increasing every day and everywhere. To improve the performance of WLANs, the use of wider channels, compared with a single or a basic 20-MHz channel, has been recently considered. This technique is commonly known as channel bonding [1].

The use of channel bonding in WLANs was introduced in the IEEE 802.11n amendment [2], where two basic 20-MHz channels can be aggregated to obtain a 40-MHz channel. The IEEE 802.11ac amendment [3] further extends this feature by allowing the use of 80- and 160-MHz channels by grouping

four and eight basic channels, respectively. It is expected that future WLAN amendments, such as IEEE 802.11ax, will continue to develop the use of wider channels [4].

However, the use of channel bonding also increases the probability that WLANs operating in the same area will overlap (i.e., two WLANs overlap if they share at least one basic channel), which may cause severe performance degradation for some or all of them. This performance degradation is caused by the coupled dynamics that occur between the overlapping WLANs due to the *listen-before-talk* characteristic of the carrier sense multiple access with collision avoidance (CSMA/CA) protocol. This effect may be particularly relevant in urban areas, where the high density of WLANs may impact the suitability of this approach.

To better understand the coupled dynamics during the operation of overlapping WLANs using channel bonding and to evaluate their effects in terms of performance, we model the described scenario using a continuous-time Markov network (CTMN) [5]. We show that the CTMN model is able to accurately capture the operation and the achievable throughput of each WLAN, despite considering a continuous backoff timer instead of the slotted backoff counter that is used in the IEEE 802.11 distributed coordination function (DCF). Note that models of the DCF that assume that all nodes are able to listen to all transmissions from other nodes, such as the model presented in [6], are not valid for the scenarios considered in this paper because this requirement does not hold in general.

The contributions of this paper are as follows.

- 1) We introduce a CTMN model that captures the coupled dynamics of multiple overlapping nonsaturated WLANs. It allows for configuring at each node the traffic load, the packet size, the backoff contention window (CW), the channel position and width, and the transmission rate.
- 2) To improve the computational efficiency when solving the CTMN model, we reduce its number of states by aggregating the activity of all nodes that belong to the same WLAN. We refer to it as the WLAN-centric model.
- 3) We describe the model and categorize the interactions that occur between multiple overlapping WLANs, as well as capture their coupled operation using the WLAN-centric model. We also show that some of the interactions are similar to those that appear in single-channel CSMA/CA multihop networks.
- 4) We formulate the optimal proportional fair channel allocation for WLANs when they use channel bonding, which gives us the upper-bound performance for a group of overlapping WLANs in saturated conditions.

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- 5) We numerically evaluate the performance achieved by a group of neighboring WLANs that use channel bonding as a function of the number of overlapping WLANs, the number of available basic channels, and the set of channel widths when WLANs randomly choose both the channel center frequency and the channel width. We then compare the results with those obtained using the proposed optimal proportional fair channel allocation scheme.

This paper is structured as follows. First, we introduce some related work in Section II. In Section III, we describe the system model and all of the assumptions that are made. In Section IV, we present and validate the analytical model. Section V characterizes the potential interactions between WLANs. It also describes the extension of the node-centric throughput model to a WLAN-centric model to improve the computational efficiency when solving it and provide a more compact characterization of the overall system as well. In Section VI, we introduce both the centralized and decentralized channel allocation schemes considered in this work. In particular, for the centralized case, we propose a waterfilling algorithm for allocating channels to a group of overlapping WLANs, as well as the hypothesis that results in the optimal proportional fair allocation. We present the results in Section VII, studying the effect that the quantity of available basic channels and of WLANs has on the system performance. Finally, the most important results of this paper are summarized in the conclusions, and several recommendations about the use of channel bonding in next-generation WLANs are provided.

## II. RELATED WORK

Since most previous studies only focused on channel bonding, channel selection algorithms, or continuous-time CSMA/CA throughput models, we present the related work in three separate sections. To the best of our knowledge, only [7] and [8] simultaneously consider the channel center frequency and channel width selection.

### A. Channel Bonding

The performance gains and drawbacks of channel bonding in IEEE 802.11n WLANs are experimentally analyzed by Deek *et al.* in [1] and Arslan *et al.* in [7], showing that channel bonding results in 1) a lower signal-to-interference-and-noise ratio due to the reduction of the transmission power per hertz each time the channel width is doubled, 2) a lower coverage range because wider channels require higher sensitivity, 3) a greater chance to suffer from and create interference, and 4) more competition with other WLANs operating in the same area. However, they also show that channel bonding can provide significant throughput gains when those issues can be overcome by adjusting the transmission power and rate.

The same considerations as in IEEE 802.11n are valid for channel bonding in IEEE 802.11ac. However, because it extends the channel bonding capabilities of IEEE 802.11n by allowing the use of 80- and 160-MHz channels, both the negative and positive aspects are accentuated. Therefore, there is much interest in developing effective solutions at both PHY and

MAC layers to get the most benefit from channel bonding. The performance of channel bonding in IEEE 802.11ac WLANs has been investigated by simulation in [9] and [10], where both static bandwidth channel access and dynamic bandwidth channel access schemes are considered. The results presented in [9] and [10] show that channel bonding can provide significant throughput gains and corroborate the fact that these gains are severely compromised by the activity of the overlapping wireless networks. The impact of hidden nodes on the network performance in a specific scenario is evaluated in [9], where a protection mechanism based on the exchange of RTS/CTS frames is proposed. The sensitivity of the secondary basic channels and how the position of the primary basic channel affects the system performance are evaluated in [10]. However, neither [9] nor [10] present any analytical model. Finally, channel bonding for short-range WLANs is considered in [11], where the impact of other WLANs on the system performance is evaluated.

### B. Channel Selection Algorithms

Channel selection algorithms in wireless networks have been the subject of numerous investigations. The first studies on this topic focused on either centralized or distributed schemes that rely on message passing (see, for instance, [12]–[15] and references therein).

These schemes are not applicable in our case, however, because different WLANs generally have different administrative domains: Indeed, as such, they are independent and autonomous systems. Channel bonding complicates the analysis even more because different groups of basic channels are used, which potentially makes communication between WLANs more difficult.

Several solutions, based mainly on graph theory, have been proposed trying to consider these constraints on communication. This solution requires decentralized algorithms for channel selection (see [16]–[18]).

Channel selection when wider channels are used has been considered only in [7] and [8]. In [7], Arslan *et al.* propose an algorithm to dynamically select the channel center frequency and to dynamically switch between a 20- or a 40-MHz channel width to maximize the throughput. However, the authors assume that the access points (APs) are able to exchange information (i.e., the achieved throughput on each channel) or that a central authority provides such information. A decentralized algorithm is proposed in [8] to select both the channel center frequency and the channel width by sensing the interference that is caused by the other neighboring WLANs.

### C. Continuous-Time CSMA/CA Models

The use of CTMN models for the analysis of CSMA/CA networks was originally developed in [5] and was further extended in the context of IEEE 802.11 networks in [19]–[23], among others. Although the modeling of the IEEE 802.11 backoff mechanism is less detailed than that by Bianchi [6], it offers greater versatility in modeling a broad range of topologies. Moreover, the experimental results in [20] and [21] demonstrate

that CTMN models, even if stylized, provide remarkably accurate throughput estimates for actual IEEE 802.11 systems. A comprehensible example-based tutorial of CTMN models applied to different wireless networking scenarios can be found in [24].

Boorstyn *et al.* [5] introduce the use of CTMN models to analyze the throughput of multihop CSMA/CA networks and study several network topologies, including a simple chain, a star, and a ring network. Wang *et al.* [23] extend the work in [5] by considering also the fairness between the throughput achieved by each node, as well as providing several approximations with the goal of reducing the model complexity by using only local information. In addition, they relate the parameters of the CTMN model with those defined by the IEEE 802.11 standard, such as the CW and the use of RTS/CTS frames. Durvy *et al.* [19] also use CTMN models to characterize the behavior of wireless CSMA/CA networks and investigate their spatial reuse gain. Nardelly *et al.* [21] extend previous models to specifically consider the negative effect of collisions and hidden terminals. They evaluate several multihop topologies and compare the results with experimental data to show that CTMN models can be very accurate. Liew *et al.* [20] validate the accuracy of CTMN to model CSMA networks using both simulations and experimental data. They also introduce a simple but accurate technique to compute the throughput of each node based on identifying the maximum independent sets of transmitting nodes. Recently, Laufer *et al.* [22] extended such CTMN models to support nonsaturated nodes and flow-based analysis of multihop networks. Finally, the CTMN model presented in [22] is used in [25] to evaluate the performance of a vehicular video surveillance system.

### III. SYSTEM MODEL

In the following, we will say that a group of WLANs are neighbors when all of the WLANs are within the carrier sense range of the others. A WLAN may belong to several groups of neighboring WLANs, and therefore, those groups of neighboring WLANs may also interact between them through it (see Fig. 1). Table I summarizes the notation used in this paper.

#### A. Network Description

We consider a system with  $M$  WLANs spatially distributed over a certain area, where WLAN  $i$  contains  $U_i$  nodes, i.e., the AP and  $U_i - 1$  STAs. A set of  $N$  predefined basic channels is at the disposal of all  $M$  WLANs. When WLAN  $i$  is initiated, or switches to a new channel, it selects a channel  $C_i$  of width  $W_i$ , which is a contiguous subset of  $c_i = |C_i|$  basic channels. If a basic channel has a width of 20 MHz, then the width of channel  $C_i$  is given by  $W_i = 20 \cdot c_i$ . The global set of channel allocations for the  $M$  WLANs is  $\mathcal{C} = \{C_1, \dots, C_M\}$ . We say that WLANs  $i$  and  $k$  *overlap* if  $C_i$  and  $C_k$  share at least one basic channel, i.e., if  $C_i \cap C_k \neq \emptyset$ , given that both  $i$  and  $k$  are inside the carrier sense range of the other. In case two WLANs overlap, we assume that they are outside the data communication range of the other, which makes the adjacent

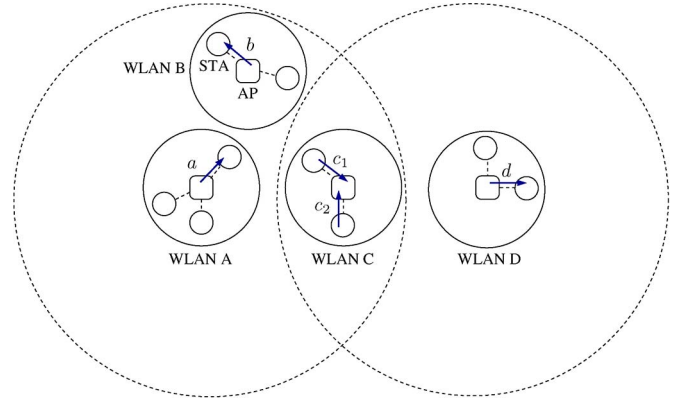


Fig. 1. Two groups of neighboring WLANs (WLANs A, B, and C on the one hand and WLANs C and D on the other). The data communication range (continuous line) and the carrier sense range (dashed line) are indicated in the plot. The two groups of neighboring WLANs interact because WLAN C belongs to both of them. Nodes  $a$ ,  $b$ ,  $c_1$ ,  $c_2$ , and  $d$  are transmitting a data flow.

channel interference negligible. Finally, we also assume that the propagation delay between any pair of nodes is zero.

#### B. Node Operation

The traffic load of node  $j$  in WLAN  $i$  is  $\alpha_{i,j}$  packets/s. When a node has a packet ready for transmission, it checks the state of the channel  $C_i$  that it has allocated. Once the channel has been sensed as being free for the duration of a Distributed InterFrame Space (DIFS), the node starts the backoff procedure by randomly initializing a timer. Every time a portion of the channel is detected as busy during the backoff interval, the backoff countdown is frozen until the entire channel width  $W_i$  is detected as free again for the duration of a DIFS interval. This counter is decremented until it reaches zero, at which time the node starts transmitting a packet using the entire channel width  $W_i$ . Note that all nodes belonging to a group of neighboring WLANs will defer their backoff countdown accordingly if they share at least a basic channel with the transmitting node. Fig. 2 shows the operation of the channel access for the specific case in which the target node uses four basic channels.

We assume that the backoff countdown at each node is in continuous time and has an average duration of  $E[B_{i,j}]$  s for node  $j$  in WLAN  $i$ . Therefore, when node  $j$  has packets waiting for transmission, the attempt rate for every node is equal to  $\lambda_{i,j} = E[B_{i,j}]^{-1}$ .

The duration of a transmission of a packet by node  $j$  in WLAN  $i$  is denoted by  $T_{i,j}(c_i, \gamma_{i,j}, L_{i,j})$  and depends on the number  $c_i$  of basic channels used, on the signal-to-noise ratio (SNR) observed at the receiver side for that transmission, i.e.,  $\gamma_{i,j}$ , and on the payload size  $L_{i,j}$ . Therefore, the packet departure rate, i.e., the rate at which packets depart from a node, is  $\mu_{i,j} = E[T_{i,j}(c_i, \gamma_{i,j}, L_{i,j})]^{-1}$ . The probability that a packet is successfully received is  $\eta_{i,j}$ . We assume that the maximum number of retransmissions per packet is infinite. In this case, the effective number of packets per second that node  $j$  has to transmit to successfully deliver its traffic load is  $\alpha'_{i,j} = \alpha_{i,j} / \eta_{i,j}$ .



of nodes that can simultaneously transmit, i.e., such that the WLANs to which they belong do not overlap. Let  $\Omega(\mathcal{C})$  be the collection of all feasible network states. Note that any change in the global channel allocation  $\mathcal{C}$  results in a different collection  $\Omega(\mathcal{C})$  of feasible network states.

Denote by  $u_{i,j}$  node  $j$  in WLAN  $i$ , with  $j = 1, \dots, U_i$ . The local dynamics at every node described in the previous section imply that the backoff rate of node  $u_{i,j}$  is  $\rho_{i,j}\lambda_{i,j}$ , with  $\rho_{i,j}$  as the long-run stationary probability that node  $j$  in WLAN  $i$  has packets ready for transmission when channel  $C_i$  is sensed empty, and therefore, the node is decreasing its backoff counter. The transmission rate of node  $u_{i,j}$  is  $\mu_{i,j} = 1/E[T_{i,j}(c_i, \gamma_{i,j}, L_{i,j})]$ . Then, the transition rates between two network states  $s, s' \in \Omega(\mathcal{C})$  are

$$q(s, s') = \begin{cases} \rho_{i,j}\lambda_{i,j}, & \text{if } s' = s \cup \{u_{i,j}\} \in \Omega(\mathcal{C}) \\ \mu_{i,j}, & \text{if } s' = s \setminus \{u_{i,j}\} \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Denote by  $S_t \in \Omega(\mathcal{C})$  the network state at time  $t$ . Due to the assumption on the backoff and transmission durations,  $(S_t)_{t \geq 0}$  is a continuous-time Markov process on the state space  $\Omega(\mathcal{C})$ . This Markov process is aperiodic, irreducible, and, thus, positive recurrent, since the state space  $\Omega(\mathcal{C})$  is finite. Hence, it has a stationary distribution, which we denote by  $\{\pi_s\}_{s \in \Omega(\mathcal{C})}$ .

Let  $\theta_{i,j}$  be the *activity ratio* of node  $u_{i,j}$ , which is defined by

$$\theta_{i,j} := \frac{\rho_{i,j}\lambda_{i,j}}{\mu_{i,j}} = \frac{\rho_{i,j}E[T_{i,j}(c_i, \gamma_{i,j}, L_{i,j})]}{E[B_{i,j}]}$$

Note that  $\theta_{i,j}$  depends on the number of basic channel  $c_i$  assigned to WLAN  $i$ , since  $\mu_{i,j}$  does. The process  $(S_t)_{t \geq 0}$  has been proven to be a time-reversible Markov process in [31]. In particular, detailed balance applies, and the stationary distribution  $\{\pi_s\}_{s \in \Omega(\mathcal{C})}$  of the process  $(S_t)_{t \geq 0}$  can be expressed as a product form. The detailed balance relation for two adjacent network states, i.e.,  $s$  and  $s \cup \{u_{i,j}\}$ , reads

$$\frac{\pi_{s \cup \{u_{i,j}\}}}{\pi_s} = \frac{\rho_{i,j}\lambda_{i,j}}{\mu_{i,j}} = \theta_{i,j}. \quad (2)$$

This relation implies that for any  $s \in \Omega(\mathcal{C})$

$$\pi_s = \pi_\emptyset \cdot \prod_{u_{i,j} \in s} \theta_{i,j} \quad (3)$$

where  $\emptyset$  denotes the network state where none of the nodes are transmitting. The last equality, together with the normalizing condition  $\sum_{s \in \Omega(\mathcal{C})} \pi_s = 1$ , yields

$$\pi_\emptyset = \frac{1}{\sum_{s \in \Omega(\mathcal{C})} \prod_{u_{i,j} \in s} \theta_{i,j}} \quad (4)$$

and

$$\pi_s = \frac{\prod_{u_{i,j} \in s} \theta_{i,j}}{\sum_{s \in \Omega(\mathcal{C})} \prod_{u_{i,j} \in s} \theta_{i,j}}, \quad s \in \Omega(\mathcal{C}). \quad (5)$$

Note that the normalizing constant  $\pi_\emptyset$  and the stationary distribution  $\{\pi_s\}_{s \in \Omega(\mathcal{C})}$  depend on the state space  $\Omega(\mathcal{C})$ , and hence, they implicitly depend on the global channel allocation  $\mathcal{C}$ .

Since the process  $(S_t)_{t \geq 0}$  is irreducible and positive recurrent on  $\Omega(\mathcal{C})$ , it follows from classical Markov chain results that  $\pi_s$

is equal to the long-run fraction of time the system spends in the network state  $s \in \Omega(\mathcal{C})$ .

### B. Packet Errors, Hidden Nodes, and External Interferers

Packets can be received with errors. Errors are generally caused by the presence of ambient noise and interference. The sources of interference are diverse. We can define two main categories based on the use or not of the CSMA/CA rules by the interferer. If the interferer is operating under the CSMA/CA rules, we will refer to it either as a contender (i.e., the interferer is inside the carrier sense range of the transmitter) or as a hidden node (i.e., the interferer is outside the carrier sense range of the transmitter). Otherwise, we will simply classify it as an external interferer.

The characterization of the interference created by hidden nodes is complicated because of the coupled dynamics with the other nodes in the network, also including that which suffers from the interference. In case of an external interferer, to characterize it, we simply require its activity pattern. Assuming that all those sources of errors are independent between them, we can define the probability that a packet transmitted by node  $j$  in WLAN  $i$  is successfully received as

$$\eta_{i,j} = (1 - p_{i,j}(\gamma_{i,j})) (1 - p_{i,j}^h) (1 - p_{i,j}^{\text{ext}}) \quad (6)$$

where  $p_{i,j}(\gamma_{i,j})$  is the probability that a packet is corrupted due to ambient noise,  $p_{i,j}^h$  is the probability that a packet is corrupted by a hidden node, and  $p_{i,j}^{\text{ext}}$  is the probability that it is corrupted by an external interferer.

There are several works that already consider the analysis of hidden nodes in Markov-based CSMA/CA network analysis (e.g., see [21] and [32] for further details).

### C. Performance Metrics

From the stationary distribution, we compute the following performance metrics.

- **Throughput:** The throughput  $x_{i,j}(\mathcal{C})$  of node  $j$  in WLAN  $i$  for a given channel allocation  $\mathcal{C}$  is

$$x_{i,j}(\mathcal{C}) := \eta_{i,j} E[L_{i,j}] \mu_{i,j} \left( \sum_{s \in \Omega(\mathcal{C}): u_{i,j} \in s} \pi_s \right). \quad (7)$$

- **Proportional fairness:** The proportional fairness of the current channel allocation with respect to the throughput is

$$f(\mathcal{C}) := \sum_{i=1}^M \sum_{j=1}^{U_i} \log x_{i,j}(\mathcal{C}). \quad (8)$$

- **Jain's fairness index:** Jain's fairness index (JFI) of the current channel allocation with respect to the throughput is

$$\mathcal{J}(\mathcal{C}) := \frac{\left( \sum_{i=1}^M \sum_{j=1}^{U_i} x_{i,j}(\mathcal{C}) \right)^2}{\left( \sum_{i=1}^M U_i \right) \left( \sum_{i=1}^M \sum_{j=1}^{U_i} x_{i,j}^2(\mathcal{C}) \right)}. \quad (9)$$



TABLE II  
VALIDATION OF THE ANALYSIS IN NONSATURATION CONDITIONS. A SINGLE SIMULATION EXECUTION  
WITH A DURATION OF 1000 S IS CONSIDERED FOR EACH EXAMPLE

	Parameters			Computation		Throughput per node [Mbps]	
	Node	$\alpha L$ [Mbps]	$E[T(c, \gamma, L)]$ [msecs]	$p(\gamma)$	$\rho$	Analysis	Simulation
Example 1	<i>a</i>	18	0.1790	0.01	0.3673	18.00	17.97
	<i>b</i>	8	0.2070	0.10	0.3662	8.00	7.98
	<i>c</i> <sub>1</sub>	10	0.2150	0.05	0.6466	10.00	9.99
	<i>c</i> <sub>2</sub>	22	0.1790	0.02	1.0000	15.95	15.75
	<i>d</i>	12	0.2630	0.15	0.6333	12.00	11.99
Example 2	<i>a</i>	4	0.1790	0.10	0.0744	4.00	3.9
	<i>b</i>	12	0.2070	0.10	0.3845	12.00	12.00
	<i>c</i> <sub>1</sub>	20	0.2150	0.15	1.0000	11.18	11.06
	<i>c</i> <sub>2</sub>	5	0.1790	0.20	0.4752	5.00	5.00
	<i>d</i>	24	0.2630	0.05	1.0000	19.00	19.06

duration are exponentially distributed. The rest of the considered parameters and their values, with the exception that, in this example, we are not considering packet aggregation ( $N_a = 1$ ), are shown in the Appendix, as well as information about the simulation tool used. The results are also shown in Table II.

### G. Insensitivity

For the Markov networks considered in this work, it turns out that the stationary distribution  $\{\pi_s\}_{s \in \Omega(\mathcal{C})}$  (and thus any analytic performance measure linked to it, such as the throughput) is insensitive to the distributions of backoff countdowns and transmission times, in the sense that it depends on these only through the ratios of their averages, i.e.,  $\theta_{i,j}$ . The proof of the insensitivity result can be found in [20] and [33]. The insensitivity property is crucial since backoff and transmission times may be not exponentially distributed in a real network.

## V. INTERACTIONS BETWEEN OVERLAPPING WIRELESS LOCAL AREA NETWORKS

The goal of this section is to characterize the different existing interactions between multiple overlapping WLANs. To do this, we first simplify the node-centric analytical model described in Section IV by aggregating states. This way, we reduce the total number of resulting network states and make its resolution more efficient. Moreover, this new point of view provides a more compact description of the interactions between neighboring WLANs. Second, we categorize the different types of interactions between WLANs, discussing how they impact each other's performance. We also show that some of those interactions are similar to those that appear in single-channel CSMA/CA multihop networks (see, for example, [5] and [22]). Finally, we validate the use of the WLAN-centric analytical model by comparing its throughput predictions with the throughput values obtained from a detailed simulation of the same considered scenarios. Moreover, the presented numerical results give us some more insights about the interactions between WLANs.

### A. WLAN-Centric Throughput Analysis

Considering several WLANs with some active nodes in each results in a large number of network states, which requires large computation resources to solve the analytical model. Therefore,

to make it more efficient, we simplify here the node-centric analytical model described in the previous section by aggregating all those states in which the nodes of a given WLAN participate. We also make the following assumptions.

- 1) We assume that all nodes in WLAN  $i$  are close to each other and to the AP and that they observe similar SNR values. Therefore, they have a similar behavior from the point of view of a node belonging to another WLAN.
- 2) Considering nonsaturation conditions, it is difficult to assess if the obtained results are due to the interaction between the different WLANs or the actual traffic load configuration of each node. To avoid such uncertainty, we assume from now on that all nodes are saturated, which can be considered as a worst-case scenario and will allow us to obtain more clear conclusions. Moreover, it also simplifies the development of the WLAN-centric model.

Therefore, since the activity of each WLAN is the sum of the activity of its  $U_i$  nodes, the WLAN-centric model is built based on the following considerations.

- 1) A network state is now defined as the set of WLANs that are active simultaneously, instead of the set of nodes.
- 2) The backoff rate of a WLAN  $i$  is the sum of the backoff rates of all nodes in it, i.e.,  $\lambda_i = \sum_{j=1}^{U_i} \lambda_{i,j}$ .
- 3) The duration of a packet transmission in WLAN  $i$  is  $T_i(c_i, \gamma_i, L_i)$ , where  $\gamma_i$  is the SNR observed by all packet transmissions inside WLAN  $i$ . Similarly, all nodes in WLAN  $i$  transmit packets of size  $L_i$  and have the same probability  $\eta_i$  to receive a packet correctly.
- 4) Since all nodes in WLAN  $i$  are assumed to be saturated, WLAN  $i$  is also saturated, and  $\rho_i = 1$ .
- 5) The activity ratio of WLAN  $i$  is given by  $\theta_i = \lambda_i E[T_i(c_i, \gamma_i, L_i)]$ .

To solve the model, the same approach as presented in Section IV is considered. Moreover, because by using the WLAN-centric model we can only compute the fraction of time a WLAN is active, the performance metrics previously described are modified accordingly. In Table III, we compare both node and WLAN-centric models in terms of computational cost. Both models are executed in the same computer and using the same version of MATLAB. The number of basic channels is set to  $N = 16$ . Statistics are obtained by executing each case 200 times. Each WLAN selects a channel

TABLE III  
COMPARISON BETWEEN THE NUMBER OF STATES AND THE COMPUTATION DELAY TO OBTAIN THE STATIONARY DISTRIBUTION BETWEEN THE NODE-CENTRIC AND WLAN-CENTRIC APPROACHES

Model	Parameters			States		Agg. Throughput [Mbps]		Computation Delay [seconds]
	M	U	$W_{\max}$ [MHz]	Mean	St. Dev.	Mean	St. Dev.	Mean
Node	6	4	160	185.46	135.71	766	177	3.1
WLAN	6	4	160	30.53	9.85	774	168	0.4
Node	8	3	80	1195.4	855.9489	945	169	20.2
WLAN	8	3	80	106.0	36.4014	966	177	1.6
Node	12	2	40	20704.0	17967.0	1141.8	1646.2	395.4
WLAN	12	2	40	738.7	356.3	1124.5	176.68	14.7

width uniformly at random from the set of available channel widths  $\{20, \dots, W_{\max}\}$  MHz. The position of the selected channel within the available channels is also uniformly picked at random. The throughput values converge by increasing the number of executions, which also increases the computation delay. It can be observed that both models give the same throughput, but the node-centric model requires much more time and computational resources.

### B. Cases of Interest

To illustrate the different cases of interest, we consider three neighboring WLANs, i.e.,  $A$ ,  $B$ , and  $C$ . All three WLANs transmit packets of fixed size ( $L$ ), have the same number of nodes ( $U$ ), have backoffs with the same average duration ( $E[B] = \lambda^{-1}$ ), and use the same modulation and coding rate regardless of the number of basic channels selected by each WLAN. Therefore, if two WLANs use the same number of basic channels, the duration of a transmission is the same in both cases. Thus, for clarity, in the notation of time durations and activity ratios in this section, we will drop the subscript  $i$  (which distinguishes the WLANs) and instead explicitly write the number of basic channels  $c_i$  assigned to WLAN  $i$ .

1) *To Overlap or Not To Overlap*: In the first example, we show that in terms of throughput, the best option for all neighboring WLANs is to use nonoverlapping channels. To illustrate this, we first consider the case in which all three WLANs use the same basic channels, namely,  $C_A = C_B = C_C = \{1,2,3,4,5,6\}$ . Therefore, the set of feasible network states is  $\Omega(\mathcal{C}) = \{\emptyset, s_A, s_B, s_C\}$ . The throughput achieved by WLAN  $A$  is

$$x_A = \frac{L}{E[T(6)]} \pi_{s_A} = \frac{\frac{L}{E[T(6)]} \theta(6)}{1 + 3\theta(6)} = \frac{U\lambda L}{1 + 3\theta(6)}.$$

By symmetry, the throughput achieved by each WLAN is identical, and therefore

$$x_A = x_B = x_C = \frac{U\lambda L}{1 + 3\theta(6)}.$$

Now, consider a different scenario in which each WLAN uses two nonoverlapping channels, namely,  $C_A = \{1,2\}$ ,  $C_B = \{3,4\}$ , and  $C_C = \{5,6\}$ . For this new channel allocation, the set of feasible network states is  $\Omega(\mathcal{C}) = \{\emptyset, s_A, s_B, s_C, s_{AB}, s_{AC}, s_{BC}, s_{ABC}\}$ . In this case, each WLAN is completely independent of the others, and the network can there-

fore be modeled as three different systems. The throughputs achieved by the WLANs are again equal and are given by

$$x'_A = x'_B = x'_C = \frac{U\lambda L}{1 + \theta(2)}.$$

Therefore, using WLAN  $A$  as a reference, we can study the cases in which the achieved throughput when all WLANs overlap is better than the case in which each WLAN uses a nonoverlapping set of channels. Because  $x_A$  and  $x'_A$  have the same numerator in both cases, the case in which all WLANs overlap will be better if  $1 + 3\theta(6) < 1 + \theta(2)$ , or, equivalently,  $T(6) < T(2)/3$ . Due to the channel access protocol defined in Section III, the latter inequality will never hold, because the duration of some headers and other protocol overheads is not affected by the channel width.

2) *Performance Anomaly*: The performance anomaly in multirate WLANs is well known [34]. Due to the channel access mechanism, which is fair in terms of transmission opportunities, all nodes are able to transmit the same number of packets on average per unit of time, and therefore, the nodes that are able to transmit at a fast rate are severely affected by nodes that can only transmit at a low rate. A similar result is observed when several WLANs overlap if they are using different numbers of basic channels.

Consider three overlapping WLANs, i.e.,  $A$ ,  $B$ , and  $C$ , with the following channel allocations:  $C_A = \{1,2,3,4\}$ ,  $C_B = \{4,5\}$ , and  $C_C = \{4\}$ . Despite the different channel widths, all three WLANs achieve the same throughput, which is given by

$$x_A = x_B = x_C = \frac{U\lambda L}{1 + \theta(4) + \theta(2) + \theta(1)}$$

which confirms the performance anomaly previously described.

The performance anomaly can be solved in several ways. For instance, the WLANs that use a wider channel can be allowed to transmit larger packets, and therefore, the overall transmission duration in all WLANs is the same. Alternatively, a different backoff duration can be assigned to each WLAN to guarantee that the WLANs that use more basic channels transmit more often.

3) *Nondirect Interactions*: In this last example, we consider the case in which the performance of two WLANs that do not overlap is affected by the presence of a third WLAN. Suppose again that there are three WLANs, i.e.,  $A$ ,  $B$ , and  $C$ , and that the channel allocation is  $C_A = \{1,2,3,4\}$ ,  $C_B = \{5,6,7,8\}$ , and  $C_C = \{4,5\}$ . In this scenario, the set of feasible network states



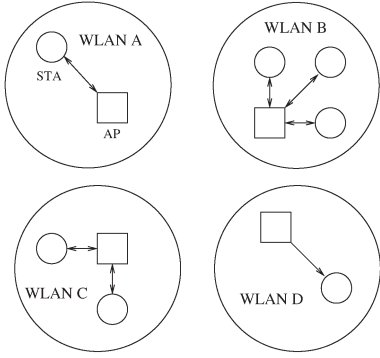


Fig. 5. Group of four neighboring WLANs. Arrows represent active traffic flows.

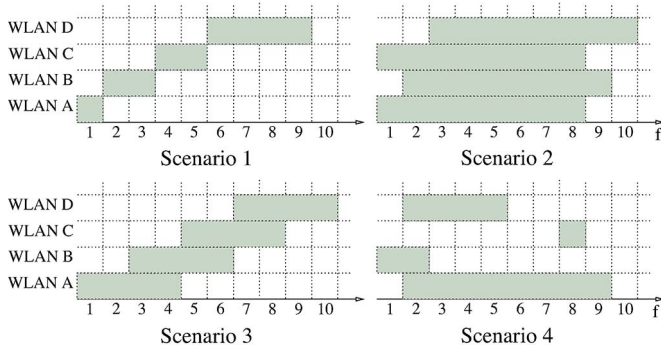


Fig. 6. Channel allocations.

is  $\Omega(\mathcal{C}) = \{\emptyset, s_A, s_B, s_C, s_{AB}\}$ . The throughput achieved by WLAN  $A$  is

$$\begin{aligned} x_A &= \frac{L}{E[T(4)]} (\pi_{s_A} + \pi_{s_{AB}}) \\ &= \frac{L}{E[T(4)]} (\theta(4x) + \theta(4)^2) \\ &= \frac{U\lambda L \cdot (1 + \theta(4))}{1 + \theta(2) + 2\theta(4) + \theta(4)^2} \end{aligned}$$

and, because  $x_B = x_A$  by symmetry, the throughput achieved by WLANs B and C is

$$\begin{aligned} x_B = x_C &= \frac{U\lambda L \cdot (1 + \theta(4))}{1 + \theta(2) + 2\theta(4) + \theta(4)^2} \\ x_C &= \frac{U\lambda L}{1 + \theta(2) + 2\theta(4) + \theta(4)^2}. \end{aligned}$$

WLAN  $A$  benefits from the existence of WLAN  $B$ , and *vice versa*, because they implicitly cooperate to starve WLAN  $C$  in the competition for the channel resources. WLAN  $C$  can only transmit when WLANs  $A$  and  $B$  are both silent.

### C. Numerical Example

Let us consider the network that is composed of four neighboring WLANs shown in Fig. 5 and the four different channel allocations shown in Fig. 6, which represent the nonoverlapping (Scenario 1), fully overlapping (Scenario 2), WLAN in the

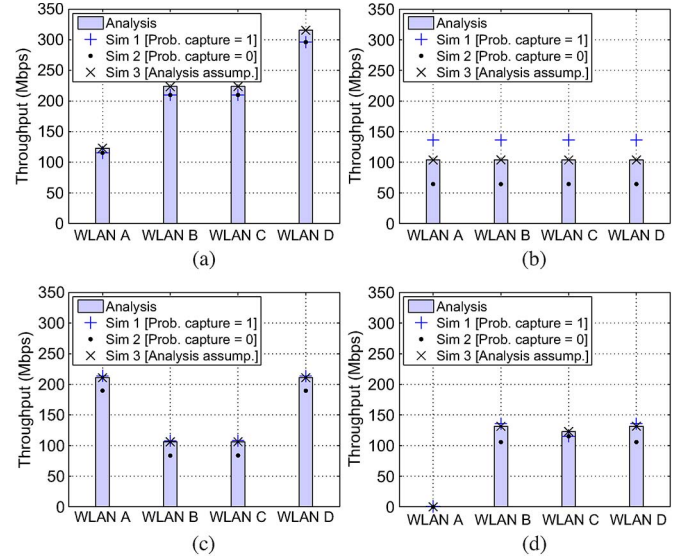


Fig. 7. Throughput achieved by each WLAN when all of them have two active nodes (i.e., the AP and one STA) in the four channel allocations considered. Each simulation result comes from a single simulation run of duration 10 000 s.

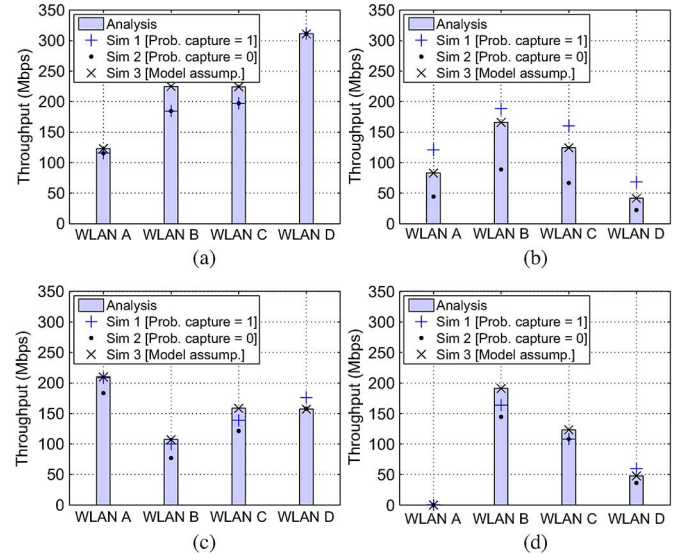


Fig. 8. Throughput achieved by each WLAN in the four channel allocations considered when WLAN  $A$  has one active STA, WLAN  $B$  has three active STAs, WLAN  $C$  has two active STAs, and, in WLAN  $D$ , only the AP is transmitting packets. Each simulation result comes from a single simulation run of duration 10 000 s.

middle (Scenario 3), and random channel selection (Scenario 4) scenarios, respectively. The number of available basic channels is set to  $N = 10$ .

The throughput achieved by each WLAN is plotted in Fig. 7 (all WLANs have two active nodes: the AP and one STA) and Fig. 8 (each WLAN has a different number of active STAs, exactly as shown in Fig. 5). Comparing these two cases allows us to visualize the effect of a different number of active STAs in each WLAN on the system performance and to determine if modeling the aggregated operation of a WLAN instead of the operation of every node is a valid approach.

The throughput for the scenario in which all WLANS have the same number of nodes and the scenario in which they do not are shown in Figs. 7 and 8, respectively. Four curves are plotted for each WLAN: the throughput computed using the WLAN-centric analytical model (bars), the throughput obtained from the simulator when the capture effect is considered (Sim 1), the throughput from the simulator when the capture effect is not considered (Sim 2), and the throughput from the simulator when the same assumptions used for the analysis are considered (Sim 3).

The results of the throughput model and Sim 3 match perfectly, which validates again the correctness of the results and shows that the insensitivity property indeed holds. Since the throughput model does not allow two or more nodes to simultaneously transmit, it does not benefit from concurrent packet receptions when the capture effect is enabled. Therefore, in some cases, when the number of overlapping WLANS is high, the capture effect causes a higher throughput than the model (Sim 1). Otherwise, if packet capture is not considered, the achieved throughput is lower than the predicted throughput by the analytical model due to the negative effect of collisions (Sim 2). The impact of each of the four channel allocations is discussed next.

Fig. 7 shows the throughput achieved by each WLAN in the four scenarios. In Scenario 1, the WLANS do not overlap because they use different groups of basic channels. Therefore, the throughput achieved by each WLAN only depends on the number of basic channels it uses. In Scenario 2, all WLANS overlap because they all use eight basic channels. In this case, all WLANS compete with all of the others for the channel, which results in the same throughput for all of them. A comparison of the results of Scenarios 1 and 2 indicates that unless the packet capture effect is enabled, using a single basic channel is better than using eight basic channels if there is overlap with the other three neighboring WLANS. In Scenario 3, the channels of WLANS B and C are located between WLANS A and D, and they all use four basic channels. This situation benefits WLANS A and D because they only overlap with WLANS B and C, respectively, which are also competing for the channel resources. Finally, Scenario 4 represents a random channel allocation. It is remarkable that WLAN A, which uses more basic channels, achieves nearly zero throughput. This occurs because it has to compete with the other three WLANS, which are in two independent groups that do not compete. WLANS B and D have the same throughput despite using different channel widths due to the performance anomaly.

Fig. 8 shows the throughput achieved by each WLAN in the same four scenarios as in Fig. 7 but with different numbers of active STAs in each WLAN. WLAN A has a single STA, WLAN B has three STAs, WLAN C has two STAs, and only the AP transmits in WLAN D. Increasing the number of STAs in a WLAN is equivalent to increasing its activity factor  $\theta$ , which also affects its throughput and how it interacts with the other networks. It is worth mentioning that a similar effect would be achieved by keeping the number of nodes per WLAN constant but reducing the backoff duration.

In terms of fairness, we compute the JFI with respect to the throughput achieved by each WLAN. The results are shown in

TABLE IV  
JFI

Scenario	Same Number of Nodes	Different Number of Nodes
1	0.9135	0.91643
2	1	0.83333
3	0.90167	0.94987
4	0.75199	0.60826

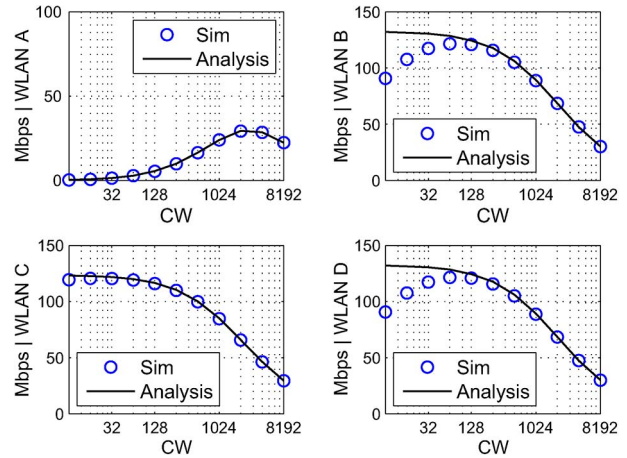


Fig. 9. Throughput achieved by each WLAN in Scenario 4 when each one has two active nodes. The probability of capturing a packet in case of collisions is set to 0, and therefore, we are considering the worst case in terms of the negative effect of collisions.

Table IV, where a low JFI value indicates that the four WLANS achieve very different throughputs.

Fig. 9 shows the throughput achieved by each of the four WLANS in Scenario 4 when the CW increases from 8 to 8192. We consider the case in which WLANS have two nodes active. The continuous-time backoff mechanism is able to capture the same dynamics as when a discrete backoff mechanism (i.e., as in IEEE 802.11 WLANS) is considered. Only when the effect of collisions is significant can the continuous-time backoff mechanism offer optimistic results. Moreover, it can be observed that almost exact values are achieved in both cases when the CW value is optimal for the discrete backoff scheme (i.e., the CW value that maximizes the throughput) since it is the value at which the negative effect of collisions becomes marginal. In addition, Fig. 9 also shows that increasing the CW value reduces the starvation suffered by WLAN A. The downside is that we severely reduce the throughput achieved by the other three WLANS.

## VI. CHANNEL ALLOCATION SCHEMES

Neighboring WLANS operating in the Industrial, Scientific and Medical (ISM) band may belong to different administrative domains, and therefore, they may select the channel to use autonomously and, in most of the cases, with no information about the current spectrum occupancy. This situation is equivalent to selecting the channel to be used uniformly at random by each WLAN.

Here, we describe such random channel selection approach, considering two channelization cases: 1) Any group of basic

channels can be selected, and 2) only the channels specified by the IEEE 802.11ac amendment can be selected. To determine the network capacity that is lost because of the absence of a controlled channel allocation, we also introduce an optimal centralized proportional fair channel allocation strategy.

### A. Decentralized Approaches

Channel allocation in autonomous WLANs is done in a decentralized way. That is, each WLAN chooses the group of basic channels to use independently. In this category, we consider two cases.

1) *Random Channel Selection*: In this scheme, WLAN  $i$  uniformly selects  $c_i$  consecutive basic channels at random from the  $N$  available basic channels.

2) *IEEE 802.11ac Channelization*: IEEE 802.11ac channelization tries to prevent WLANs using the same number of basic channels from partially overlapping. This is achieved by explicitly defining the groups of basic channels that can be selected when  $c_i$  channels are going to be used. That is, given that a WLAN is going to use  $c_i$  basic channels, it can only select  $\lfloor N/c_i \rfloor$  different channels. Once one of these channels is selected, the first basic channel in it is  $c_i(Z_1 - 1) + 1$ , and the last basic channel is  $c_i Z_1$ , where  $Z_1 = \mathcal{U}([1, \dots, \lfloor (N/c_i) \rfloor])$  is a uniformly distributed random value between 1 and  $\lfloor (N/c_i) \rfloor$ . Note that the available basic channels are numbered from 1 to  $N$ .

### B. Centralized Approach

When all WLANs are mutually within the carrier sense range, we characterize the optimal *proportional fair channel allocation* as a linear combination of *waterfilling solutions*, assuming that WLANs can periodically alternate between different allocations. This optimal allocation is the best tradeoff between maximizing throughput and fairness, in the sense that, starting from the optimal allocation, a proportional increase of the throughput for any set of WLANs would result in a bigger proportional decrease of throughput for the remaining WLANs.

Then, we will show how to relax the two assumptions we made in the following way.

- When not all WLANs are mutually within the carrier sense range, we present a technique to devise a suboptimal solution.
- If the WLANs cannot periodically alternate between different allocations, we show that a single waterfilling solution is a reasonable suboptimal choice.

This is an idealized approach, where a central server with knowledge of the WLAN topology is needed. However, the computation required only depends on the number of WLANs that mutually interfere and on the number of basic channels available. Moreover, it is easy to compute in an efficient way, and such computation can be done preemptively.

1) *Proportional Fair Channel Allocation*: Let  $\mathcal{K}$  be the collection of all possible sets of channels, i.e.  $\mathcal{C} \in \mathcal{K}$ . We call  $x(\mathcal{C})$  the corresponding aggregate throughput of a set. For example, each of the four channel allocations represented in Fig. 6 has

a different  $\mathcal{C} \in \mathcal{K}$  and a corresponding aggregate throughput  $x(\mathcal{C})$ .

We want to characterize the optimal  $\mathcal{C} \in \mathcal{K}$ . However, this problem is, in general, hard to solve because of the combinatorial structure of the discrete collection of sets  $\mathcal{K}$ . To simplify the analysis, we need to allow WLANs to switch their channel configuration  $\mathcal{C}$  at any time and look for an optimal time schedule for the network along a time period. In other words, we allow WLANs to switch to a different  $\mathcal{C}$  and keep that configuration for a certain period of time.

We define a (*global*) *schedule*  $p(\mathcal{C}) : \mathcal{K} \mapsto [0,1]$  as the portion of time the network spends on each channel configuration  $\mathcal{C}$ . Since we are including all the possible channel configurations in  $\mathcal{K}$ , including those in which some WLANs are not transmitting at all (i.e.,  $C_i = \emptyset$ ), the schedule vector must sum to one, i.e.,  $\sum_{\mathcal{C} \in \mathcal{K}} p(\mathcal{C}) = 1$ .

For example, considering Fig. 6, a possible (although clearly not optimal) schedule would be that in which the system uses Scenario 1 for half of a time period, and no WLAN is transmitting ( $C_i = \emptyset$  for all  $i$ ) the rest of the time.

To determine the proportional fair global scheduling, we need to solve the following utility optimization problem.

*Problem 1 (Proportional Fairness)*:

$$\begin{aligned} & \max_p \sum_{i=1}^M \log \sum_{\mathcal{C} \in \mathcal{K}} p(\mathcal{C}) x_i(\mathcal{C}) \\ & \text{s.t.} \sum_{\mathcal{C} \in \mathcal{K}} p(\mathcal{C}) = 1 \\ & p(\mathcal{C}) \in [0,1], \quad \text{for all } \mathcal{C} \in \mathcal{K}. \end{aligned}$$

The quantity  $\sum_{\mathcal{C} \in \mathcal{K}} p(\mathcal{C}) x_i(\mathcal{C})$  is the throughput achieved by WLAN  $i$  using the schedule  $p(\mathcal{C})$ , and it is computed as the weighted average of the different throughputs  $x_i(\mathcal{C})$  for the various  $\mathcal{C} \in \mathcal{K}$ .

a) *Properties of Problem 1*: This problem requires the maximization of a concave function in a convex set; thus, it is easy to solve in principle. The objective function is concave because when  $p(\mathcal{C})$  is a vector with  $|\mathcal{K}|$  entries,  $f_i(p(\mathcal{C})) = \sum_{\mathcal{C} \in \mathcal{K}} p(\mathcal{C}) x_i(\mathcal{C})$  is affine; hence,  $\log f_i(p(\mathcal{C}))$  is concave because it is composed of an affine function, and the sum of concave functions is concave. Moreover, the constraints are clearly convex. This formulation is broad enough to include the case when not all WLANs are mutually within the carrier sense range.

Unfortunately, the size of  $\mathcal{K}$  exponentially grows with  $M$ , which makes the computation of the throughput function  $x(\cdot)$  challenging. To overcome this issue, we will now characterize more in detail the optimal and suboptimal solutions, to be able to derive them without explicitly solving the convex problem.

b) *Waterfilling solution*: We can define the waterfilling solution when all WLANs are mutually within the carrier sense range. We will relax this assumption later. Given the number of basic channels  $N$ , we can easily build a mapping from the number of WLANs  $M$  to the allocation that minimizes the number of overlaps between WLANs.

Algorithm 1 shows the pseudocode to build such a mapping,  $f(M, N)$  (similarly as [35] for the case of the free-disposal property). The number of basic channels used is doubled once for each WLAN until the number of available basic channels allows it. The first channel positions are then chosen such that the spectrum is evenly used.

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**Algorithm 1** Waterfilling Algorithm
 

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1: assign to each WLAN a single basic channel, i.e.,  $c_i = 1$ 
   for all  $i = 1, \dots, M$ .
2: loop
3:   for  $i = 1, \dots, M$  do
4:     if  $2c_i + \sum_{j \neq i} c_j \leq N$  then
5:        $c_i \leftarrow 2c_i$ 
6:     else
7:       goto 11
8:     end if
9:   end for
10: end loop
11: For each WLAN  $i$ , select the basic channels as the contiguous set  $[1 + \sum_{j < i} \min(c_j, W_{\max}), \sum_{j \leq i} \min(c_j, W_{\max})]$  modulo  $N$ .
    
```

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This procedure always produces an allocation that minimizes the number of overlaps per channel. Moreover, the obtained allocation is such that either all WLANs have the same width or there are only two sets of widths. In the latter case, we can split the WLANs into two sets  $G_1$  and  $G_2$ , such that  $c_i = 2 \cdot c_j$  for each  $i \in G_1, j \in G_2$ .

Such waterfilling allocation plays a key role in the proportional fair allocation, even in the relaxed cases of when not all WLANs are mutually within the carrier sense range and when WLANs cannot periodically alternate between different allocations; we show in the following that a single waterfilling solution is a good suboptimal choice.

*c) Proportional fairness and waterfilling:* We now present a conjecture regarding the relationship between the waterfilling configuration and the proportional fair configuration.

*Conjecture 1:* The proportional fair solution to Problem 1 with the throughput function that was defined in Section IV is a linear combination on waterfilling configurations only.

This means that to have a proportional fair configuration, the WLANs should change roles in turn between the (nonunique) waterfilling solutions.

If such a time-slicing function is not available, any solution from the waterfilling configuration is an acceptable suboptimal solution. We corroborate this claim by means of simulation in Section VII (see Fig. 15 in particular).

We simulated different scenarios and solved Problem 1 using the MATLAB CVX framework. Conjecture 1 was never conuted in our simulations.

*d) Interactions between multiple groups of neighboring WLANs:* We present a technique to devise a suboptimal solution when not all WLANs are mutually within the carrier sense range. We need such a technique because, although Problem 1 would still represent such scenarios and would still be convex,

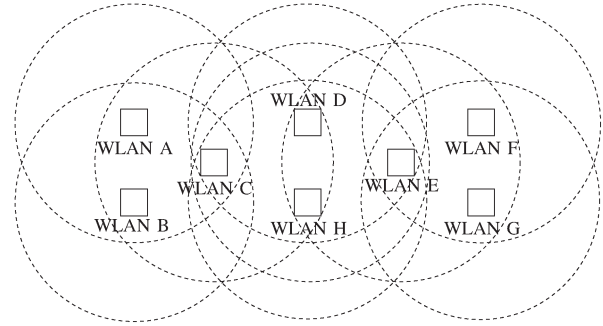


Fig. 10. Eight WLANs distributed in four groups of neighboring WLANs: WLANs A, B, and C in group 1; WLANs C, D, and H in group 2; WLANs D, E, and H in group 3; and WLANs E, F, and G in group 4.

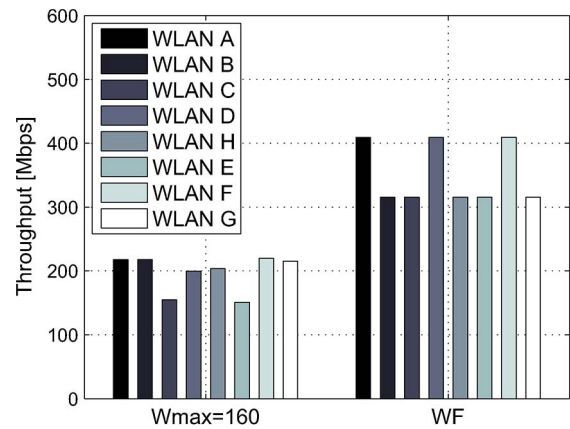


Fig. 11. Expected throughput achieved by each WLAN in the scenario shown in Fig. 10. Random channel allocation versus waterfilling algorithm.

Conjecture 1 is no longer valid. Consequently, characterizing the optimal solution becomes very hard in general.

First, we need to consider the interference graph of the network  $G = (V, E)$ , where  $V$  is the set of WLANs, and the edges are defined as  $e = (i, j) \in E$  if WLAN  $i$  can interfere with WLAN  $j$ . Interference is assumed to be symmetric, and thus,  $G$  is an undirected graph.

We can compute the chromatic number  $\chi$  of this graph, i.e., the minimal number of colors necessary to have the property that no neighbors share the same color. A coloring with  $\chi$  colors represents an equivalence relation of minimum cardinality such that all WLANs that share the same color do not interfere and, thus, can choose the same set of channels.

Therefore, we can consider the collection of  $\chi$  groups of WLANs with the same color as a collection of virtual WLANs and use the mapping  $f(\chi, N)$  obtained using Algorithm 1 over these virtual WLANs, i.e., we use a waterfilling allocation where all WLANs that share the same color will have the same allocation.

As an example, let us consider the scenario shown in Fig. 10. The number of available basic channels is set to  $N = 19$ . WLANs select both the width and the position of the selected channel uniformly at random. The set of available channel widths is  $\{20, 40, 80, 160\}$  MHz.

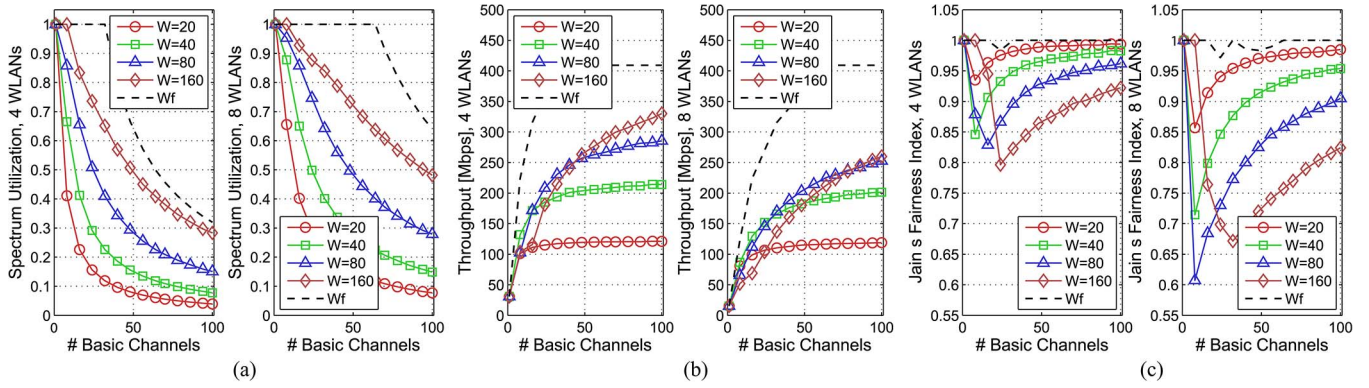


Fig. 12. Throughput, spectrum utilization, and fairness when the number of basic channels increases. (a) Spectrum Utilization. (b) Expected Throughput per WLAN. (c) Jain's Fairness Index.

In this case, we have  $\chi = 3$ , and the groups that share the same color are  $\{I_1, I_2, I_3\} = \{\{A, D, F\}, \{C, E\}, \{B, H, G\}\}$ . If we run the waterfilling algorithm on  $\{I_1, I_2, I_3\}$ , we get the same solution of a complete graph with three WLANs, such that one WLAN will have a width equal to 8 and two WLANs will have a width equal to 4. A possible solution obtained with this technique is  $C_A = \{1-8\}$ ,  $C_B = \{13-16\}$ ,  $C_C = \{9-12\}$ ,  $C_D = \{1-8\}$ ,  $C_H = \{13-16\}$ ,  $C_E = \{9-12\}$ ,  $C_F = \{1-8\}$ , and  $C_G = \{13-16\}$ , which, as shown in Fig. 11, results in a higher throughput than using the random channel allocation scheme. In case all WLANs are interfering with each other, then the set  $I$  corresponds simply with the set of WLANs.

If WLANs can alternate between channel allocations, then all WLANs will have, on average, the same throughput, alternating the roles among the color groups. However, even in this case, the solution is generally suboptimal, because a very unbalanced interference graph could allow more aggressive solutions, i.e., it could happen that some nodes belonging to a certain color group would be allowed to select a wider width than the other nodes in the same group.

## VII. PERFORMANCE EVALUATION

Here, we evaluate the impact to the system of a different number of neighboring WLANs, a different number of available basic channels, and the set of channel widths that are available for each WLAN.

### A. Increasing the Number of Channels

Fig. 12 shows the expected spectrum utilization [see Fig. 12(a)], the expected throughput of a single WLAN [see Fig. 12(b)], and the expected throughput fairness [see Fig. 12(c)] for four and eight neighboring WLANs when the number of basic channels increases from 1 to 100 and all WLANs use the same channel width  $W$ . The spectrum utilization is computed as the fraction of basic channels that are occupied by one or more WLANs versus the total number of basic channels, i.e.,

$$v(C) := \frac{1}{N} \sum_{k=1}^N I(k) \quad (10)$$

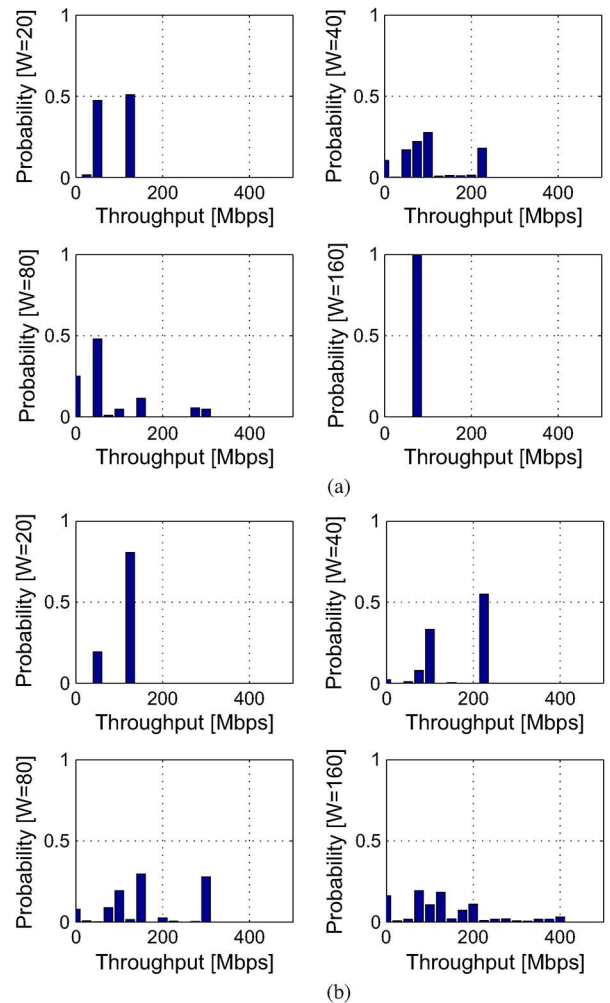


Fig. 13. Histogram of the aggregate throughput for different  $N$  and  $M$  values. (a)  $N = 8$ ,  $M = 6$ . The expected throughput values are 89.847, 103.21, 78.965, and 69.031 Mb/s for  $W = 1, 2, 4$ , and 8, respectively. (b)  $N = 24$ ,  $M = 6$ . The expected throughput values are 110.23, 166.28, 170.33, and 130.7 Mb/s for  $W = 1, 2, 4$ , and 8, respectively.

where the function  $I(k)$  returns 1 if the basic channel  $k$  is found occupied by one or more WLANs.

The results in Fig. 12 show that the waterfilling algorithm is able to maximize the spectrum utilization while distributing

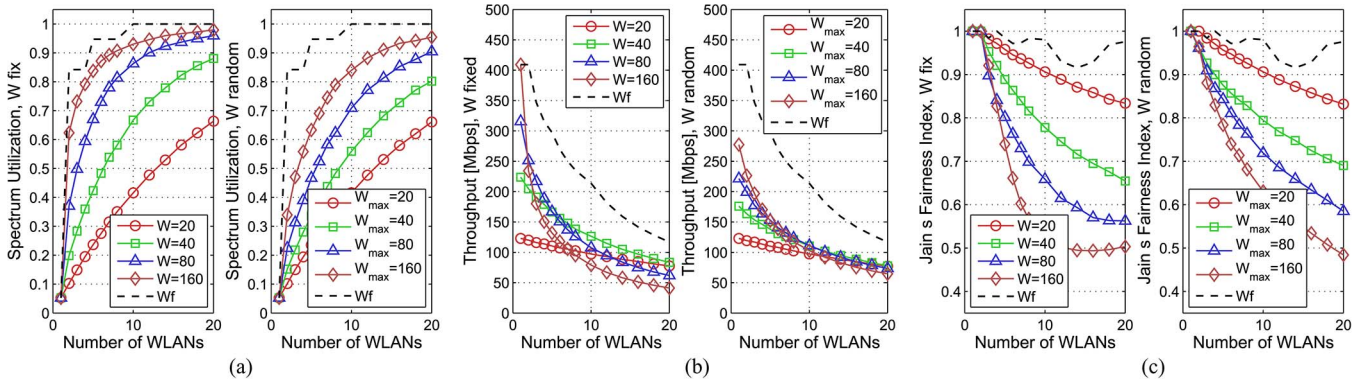


Fig. 14. Throughput, spectrum utilization, and fairness when the number of WLANs increases. (a) Spectrum Utilization. (b) Expected Throughput per WLAN. (c) Jain's Fairness Index

the available basic channels evenly among the neighboring WLANs. As a consequence, it provides the highest WLAN network throughput and fairness. The JFI values that are less than 1 are obtained when not all WLANs in the resulting channel allocation have allocated the same channel width.

When each WLAN randomizes the group of basic channels to be used without any information about the spectrum occupancy or the number of neighbors, selecting a large group of basic channels only guarantees a higher throughput when the number of neighboring WLANs is small or when the number of available basic channels is very large. For example, when there are only four neighboring WLANs, selecting  $W = 160$  MHz only gives a higher throughput than  $W = 80$  MHz if more than 50 basic channels are available. Finally, in terms of fairness, the use of a large  $W$  also accentuates the differences in the throughput achieved by each WLAN. The fairness is therefore low because most of the neighboring WLANs overlap, which results in a significantly lower throughput than the few that do not.

To obtain more insight into the system dynamics, Fig. 13 shows the histogram of the achieved throughput by a single WLAN when there are six neighboring WLANs and two numbers of basic channels:  $N = 8$  and  $N = 24$ . We used the throughput of 10 000 randomly generated scenarios to obtain these histograms. The histogram shows all of the possible throughput values and the probability of achieving each one. For  $N = 8$  [see Fig. 13(a)] and  $W = 20$  MHz, the throughput achieved by a single WLAN is higher than 100 Mb/s in approximately 50% of the cases, which corresponds to the case in which none of the WLANs overlap. Increasing the channel width to  $W = 40$  MHz increases the chances that the WLANs overlap, which reduces the expected WLAN throughput. However, in approximately 20% of the cases, the WLANs randomly select 40-MHz nonoverlapping channels, which results in a higher throughput. Similar observations can be made for  $W = 80$  MHz, where a maximum throughput of approximately 300 Mb/s can be achieved in only a few cases, as it is more likely to obtain a lower throughput than when using  $W = 40$  MHz due to the higher overlapping probability. Finally, there is a single throughput value for  $W = 160$  MHz because all WLANs overlap. Similar observations can be made for  $N = 24$  basic channels [see Fig. 13(b)]. In this case, it is clear that

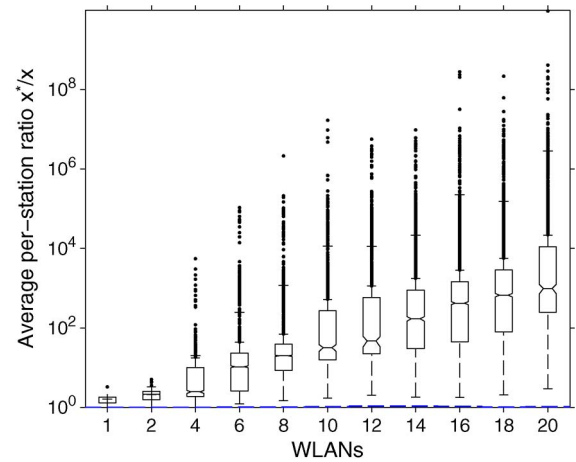


Fig. 15. Average per-WLAN ratio distribution  $(1/M) \sum_{i=1}^M (x_i^*/x_i)$  between optimal throughput and sampled throughput of random channel allocations with  $W_{\max} = 160$  MHz. Blue dotted line represents the waterfilling solution.

the presence of more basic channels allows more combinations and improves the overall system performance. In this example, the optimal value of  $W$ , which is the  $W$  value that results in the highest average throughput, is 40 MHz (103.21 Mb/s) and 80 MHz (170.33 Mb/s) for  $N = 8$  and  $N = 24$ , respectively. Note that the expected WLAN throughput achieved in each case is shown in the caption of Fig. 13.

### B. Increasing the Number of WLANs

In Fig. 14, we show the system performance when the number of neighboring WLANs increases. The number of available basic channels is set to  $N = 19$ . We also evaluate the case in which each WLAN randomly chooses the value of  $W$  given a maximum value,  $W_{\max}$  (i.e.,  $W$  is a random value that is uniformly distributed between the feasible values of 20 MHz and  $W_{\max}$ ).

The results show that increasing the number of WLANs results in a higher spectrum utilization [see Fig. 14(a)], lower throughput [see Fig. 12(b)], and generally lower fairness [see Fig. 14(c)]. Note that the effect of randomly selecting  $W$  increases the ways that different WLANs interact because more

TABLE V  
COMPARISON BETWEEN PURE RANDOM AND IEEE 802.11AC CHANNEL ALLOCATION SCHEMES.  
THE VALUE OF  $W$  IS SELECTED UNIFORMLY AT RANDOM BETWEEN 20 AND  $W_{\max}$

Parameters	Aggregate Throughput [Mbps]						JFI			
	N	M	$W_{\max}$ [MHz]			$W_{\max}$ [MHz]				
Channelisation			20	40	80	160	20	40	80	160
Random	16	8	789.1	897.6	909.2	844.0	0.95	0.95	0.93	0.91
802.11ac	16	8	794.2	936.4	966.5	928.2	0.95	0.95	0.95	0.93
Random	16	12	1058.4	1119.7	1092.9	974.0	0.96	0.95	0.92	0.89
802.11ac	16	12	1058.4	1156.6	1157.1	1081.2	0.96	0.96	0.94	0.92
Random	16	16	1264.7	1276.7	1212.2	1065.3	0.97	0.94	0.91	0.87
802.11ac	16	16	1263.0	1310.7	1288.8	1157.0	0.97	0.95	0.93	0.90

combinations are feasible, which, for any  $W_{\max}$  value and a large number of WLANs, results in a throughput similar to that achieved when  $W_{\max} = 20$  MHz. However, the fairness decreases with  $W_{\max}$ .

Fig. 15 shows the average per-WLAN ratio distribution  $(1/M) \sum_{i=1}^M (x_i^*/x_i)$  between the optimal throughput and the sampled throughput of random allocations with  $W_{\max} = 160$  MHz. The same quantity for the waterfilling solution is shown. The box represents the samples inside the interquartile range  $Q3 - Q1$ , the crosses represent the average, and the notches represent the medians. The black dots represent outliers (samples more than 1.5 times the interquartile range). Any solution that is different than the proportional fair solution makes the sum of the proportional gain negative (and thus also the average). When  $W_{\max} = 160$  MHz, rare events of starvation effect occur, particularly when the number of WLANs increases.

### C. IEEE 802.11ac Channelization

In Table V, we show the expected aggregate throughput and throughput fairness for the two decentralized channel allocation schemes considered in this work. The considered number of available basic channels is set to  $N = 16$  for a fair comparison between both schemes. Similar results between both channel selection schemes are obtained. However, since the IEEE 802.11ac channelization prevents the negative effects of partial overlaps between WLANs, and nondirect interactions as well, it results in a slightly better aggregate throughput and throughput fairness for large  $W$ .

### D. Final Remarks

Fig. 14 shows that a random channel selection greatly affects the overall system performance. For instance, with ten WLANs, the expected throughput achieved by a single WLAN using the waterfilling algorithm is almost 100% higher than the expected throughput achieved with  $W_{\max} = 40$  MHz, which is the channel width that gives the best throughput when the channel position is randomly selected. Similarly, the JFI value is significantly lower than that obtained by the waterfilling algorithm. The spectrum utilization is also lower because several WLANs use the same basic channels, whereas others remain empty. Similar observations can be made from Fig. 12.

Therefore, there is an important gap between the performance of the centralized channel allocation algorithm and the

TABLE VI  
PARAMETER VALUES BASED ON IEEE 802.11AC

Parameter	Notation	Value
Packet Length	$L_d$	12000 bits
Number of aggregated packets	$N_a$	64 packets
Number of SU-MIMO spatial streams	$N_{su}$	2 packets
Backoff Contention Window	CW	16 slots
Slot Duration	$T_{slot}$	9 $\mu$ s
Average time decreasing the backoff	$E[B]$	$\frac{CW}{2} \cdot T_{slot}$
DIFS	-	34 $\mu$ s

performance when each WLAN randomly selects the channel to use. There are several possible solutions to reduce this gap for autonomous WLANs: 1) Use a database in the cloud to store information about the channels that are used in each geographical area. This database could be used to find empty channels for new WLANs. However, there is no way to force already existing WLANs to adapt to an increasing demand in that area and reduce their channel width. 2) Use a decentralized channel selection algorithm that is able to adapt to the spectrum occupancy based on the instantaneous information that it is able to infer from the behavior of the neighboring WLANs.

## VIII. CONCLUSION

In this paper, we have introduced, described, and characterized the interactions that occur in the operation of multiple neighboring WLANs when they use channel bonding. To capture these interactions, we have developed and validated an analytical framework based on a CTMN model. This framework was then used to evaluate the system performance in terms of the number of neighboring WLANs, the number of basic channels available, and the set of channel widths that each WLAN is allowed to use. We have also proposed a centralized waterfilling algorithm that provides a proportional fair global channel allocation, or at least the best suboptimal allocation, because we are dealing with a discrete state space.

The results obtained when WLANs select the channel center frequency and the channel width randomly, which is a good representation of what occurs in real deployments, show that the throughput, the spectrum utilization, and the fairness are significantly lower than the values obtained using the centralized algorithm. This indicates the need to develop smarter decentralized channel selection algorithms to make efficient use of the available spectrum for autonomous WLANs.

TABLE VII  
TRANSMISSION RATES FOR EACH NUMBER OF BASIC CHANNELS. THESE VALUES ARE COMPUTED FOR A PACKET ERROR PROBABILITY LESS THAN 10% FOR 4096-BYTE PACKETS [3]

$c$	Data Subcarriers, $\xi(c)$	Modulation, $N_m$	Coding Rate, $N_c$	Tx Rate (Mbps)	Min. Sensitivity (dBm)
1	52	64-QAM, 6 bits	5/6	65	-64
2	108	64-QAM, 6 bits	3/4	121.5	-62
4	234	16-QAM, 4 bits	3/4	175.5	-61
8	468	16-QAM, 4 bits	1/2	232	-65

## APPENDIX

### SYSTEM PARAMETERS AND SCENARIO CONSIDERATIONS

We assume that all WLANS operate using the IEEE 802.11ac amendment [3]. Therefore, the WLANS operate in the 5-GHz ISM band, where each basic channel has a width of 20 MHz.  $W$  can take values from  $\{20, 40, 80, 160\}$  MHz. In other words, the channel  $C_i$ , which is selected by WLAN  $i$ , can be composed of  $c_i \in \{1, 2, 4, 8\}$  basic channels. The parameters considered are shown in Table VI. Moreover, we consider that for each value of  $c$ , WLANS use a different modulation and coding rate, as shown in Table VII. Unless otherwise stated, all WLANS have two nodes.

All nodes are equipped with at least two antennas, which they use to transmit two spatial streams in single-user multiple-input multiple-output (MIMO) mode. Packet aggregation is also considered, and 64 packets are included in each transmission. Under these conditions, the time required to transmit a packet by node  $j$  in WLAN  $i$  is  $T_{i,j}(c_i, \gamma_{i,j}, L_{i,j})$ , which is computed as follows:

$$\begin{aligned}
 & T_{i,j}(c_i, \gamma_{i,j}, L_{i,j}) \\
 &= \left( T_{\text{PHY}} + \left\lceil \frac{\text{SF} + N_a(\text{MD} + \text{MH} + L_{i,j}) + \text{TB}}{N_{\text{su}}L_{\text{DBPS}}(c_i, \gamma_{i,j})} \right\rceil T_s \right) \\
 &+ \text{SIFS} + \left( T_{\text{PHY}} + \left\lceil \frac{\text{SF} + L_{\text{BA}} + \text{TB}}{L_{\text{DBPS}}(1, \gamma)} \right\rceil T_s \right) + \text{DIFS} + T_{\text{slot}}
 \end{aligned} \quad (11)$$

where  $T_{\text{PHY}} = 40 \mu\text{s}$  is the duration of the PHY-layer preamble and headers,  $T_s = 4 \mu\text{s}$  is the duration of an orthogonal frequency-division multiplexing (OFDM) symbol. SF is the *service field* (16 bits), MD is the *MPDU Delimiter* (32 bits), MH is the *MAC header* (288 bits), TB is the number of *tail bits* (6 bits), and  $L_{\text{BA}}$  is the *Block-ACK* length (256 bits).  $L_{\text{DBPS}}(c, \gamma) = N_m(\gamma)N_c(\gamma)\xi(c)$  is the number of bits in each OFDM symbol, where  $N_m(\gamma)$  is the number of bits per modulation symbol,  $N_c(\gamma)$  is the coding rate, and  $\xi(c)$  is the number of data subcarriers when  $c$  basic channels are bonded together.  $N_{\text{su}} = 2$  is the number of single-user MIMO streams, and  $N_a = 64$  is the number of packets that are aggregated in each transmission.

Finally, we consider that in all WLANS, the STAs are located near the AP. In that situation, the packet error probability is assumed to be negligible.

In all the plots, unless otherwise stated, each point is the average result of 2000 different randomly generated scenarios, where each scenario represents a single generated global channel allocation  $\mathcal{C}$ . This number of realizations guarantees that the

standard deviation of the error in the sample mean relative to the true mean of the computed throughput is less than 10 Mb/s, which is considered an acceptable error.

### A. Simulation Tool

To validate the analytical model, a simulator of the described scenario was built based on the Component Oriented Simulation Toolkit libraries [36]. The simulator accurately reproduces the described scenario and the operation of each node, including the slotted backoff mechanism that is considered in IEEE 802.11 WLANS, the presence of collisions, and the capture effect when multiple packets are simultaneously received with very different power levels [27]. In the simulator, when the capture effect is enabled, we assume that collisions between packets from nodes that belong to different WLANS do not cause the loss of the transmitted packets at the corresponding receiver.

By comparing the results obtained from the simulator with those obtained from the analytical model, we can assess the model's accuracy and the impact of the assumptions that were required to construct it.

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